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Monetary financing with interest-bearing money

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Abstract

Recent results suggesting that monetary financing is more expansionary than bond financing in standard New Keynesian models rely on a duality between policy rules for the rate of money growth and the short-term bond rate, rather than a special role for money. We incorporate two features into a simple sticky-price model to generalize these results. First, that money may earn a strictly positive rate of return, motivated by recent debates on the introduction of central bank digital currencies and the introduction of interest-bearing reserves. This allows money-financed transfers to be used as a policy instrument at the effective lower bound, without giving up the ability to use the short-term bond rate to stabilize the economy in normal times. Second, a simple financial friction generates a wealth effect on household spending from government liabilities. Though temporary money-financed transfers to households can stimulate spending and inflation when the short-term bond rate is constrained by a lower bound, similar effects could be achieved by bond-financed tax cuts. So our results do not provide compelling reasons to choose monetary financing rather than bond financing.

Key words: Monetary financing, zero lower bound, interest-bearing money, digital currency.

JEL classification: E43, E52, E62.

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1 Introduction

The global financial crisis prompted macroeconomic policymakers to employ a range of unconventional policies to stabilize economic activity and inflation, as short-term policy rates hit the effective lower bound. Despite these dramatic policy actions, the protracted effects of the crisis and the potential limits to unconventional measures have prompted proposals for fiscal stimulus as a policy tool. In particular, many have argued that this stimulus should be financed by money creation, as money-financed deficits are argued to have some direct effect on spending and lead to a smaller crowding out effect via higher interest rates.

Recent research has analyzed the effects of money-financed fiscal policies using conventional New Keynesian models (Galí, 2014a; English et al., 2017), under the standard assumption that money earns no interest. In such models there is no wealth effect from a money-financed deficit even if money earns no interest and does not require higher future taxes to meet the servicing costs of conventional government debt (Weil, 1991).

We use a small sticky-price model to study the possible advantages of money-financed transfers to households, relative to conventional fiscal policy, when the economy is in a temporary liquidity trap. Our model is standard in most respects. Money alleviates transactions frictions faced by households.

Our model includes two important features that allow us to extend previous results. First, we assume that money may earn a strictly positive rate of return. This is motivated by the recent debates on the introduction of central bank digital currencies (which in some forms could be remunerated) and the introduction of interest-bearing central bank reserves in many economies following the financial crisis. Second, we incorporate a simple financial friction that implies that households regard government liabilities as net wealth (Weil, 1991; Ireland, 2005).

The first feature allows policymakers to control the stock of money independently of the nominal interest rate on short-term government bonds. In contrast, conventional monetary models assume that the rate of interest on money is zero. Monetary policy in these models is implemented *either* by a policy rule for the evolution of the money stock or by a policy rule for the rate of interest on short-term nominal bonds. The latter can be implemented by supplying whatever quantity of money is required (via open market operations in short-term bonds) to deliver the rate of interest implied by the rule. This conventional approach implies a duality between the rate of money growth and the short-term bond rate.[1] Our approach therefore allows us to compare the effects of monetary and bond-financed transfers *for a given monetary policy rule*.

The second feature of our model implies that expansions of government liabilities (money and one-period government bonds) do have the potential to increase aggregate demand through a wealth effect. This reflects the notion that real asset holdings may have a direct effect on consumption. To incorporate this effect we assume that each period households face a constant probability of a default-like event that restricts

[1]This duality is a well-known feature of conventional monetary models. A textbook analysis can be found in Woodford (2003, Chapter 2).

participation in asset markets (Castelnuovo and Nisticò, 2010; Nisticò, 2012; Del Negro et al., 2015). The presence of this friction implies that, in equilibrium, households 'over-discount' future income streams. One implication of this is that changes in real money balances can stimulate spending by increasing households' net wealth. Our implementation implies that this wealth effect also applies to bond-financed stimulus, which allows us to determine whether there are specific benefits to money-financed fiscal actions.

We use our model to demonstrate that recent proposals for money-financed fiscal stimulus (in particular, Galí, 2014a) rely on the conventional duality between the rate of money growth and the short-term bond rate. We confirm the findings of English et al. (2017), who show that the stimulative effects of such policies are determined by the fact that the money supply rule implies that the short-term bond rate responds weakly to inflation, rather than because of any special property of money itself. We further demonstrate that the macroeconomic responses to non-fiscal shocks may be undesirable when this type of monetary rule is used.

These results motivate our modeling choices. We allow a policymaker to control the money stock independently of the short-term bond rate. This allows monetary transfers to be used as a policy instrument at the effective lower bound, without giving up the ability to use the short-term bond rate to stabilize the economy in response to shocks in normal times.

We show that money-financed transfers to households can increase output and inflation when the economy is in a temporary liquidity trap. Such transfers increase household wealth and hence spending and inflation, even if they are implemented in the form of a *temporary* increase in the stock of money. We investigate the potential quantitative effects of such policies by simulating the effects of a temporary increase in the rate of money growth. We assume that monetary policy continues to be governed by a rule for the short-term bond rate. So for households to willingly hold the additional money balances, the policymaker must increase the rate of interest on money.

However, our results reveal three reasons to be cautious about the use of money-financed transfers to stimulate the economy at the lower bound. First, the scale of the monetary transfers required to deliver a meaningful increase in aggregate demand and inflation is likely to be extremely large. Second, the frictions in our model suggest that equivalent stimulatory effects could be achieved by an increase in government debt, without requiring interest-bearing money. Finally, the stimulative effect of money-financed transfers is likely to be sensitive to the precise nature of the frictions that give rise to a meaningful role for money and the policy rule used to set the short-term bond rate.

We contribute to the recent debate on the potential efficacy of money-financed policy measures in a liquidity trap. Many recent contributions refer to the idea of 'helicopter drops', named for Milton Friedman's famous thought experiment in which "a helicopter [...] drops an additional $1,000 in bills from the sky".[2] These

[2]Friedman (1969).

contributions follow Bernanke (2002) and interpret Friedman's thought experiment as "essentially equivalent" to a money-financed tax cut. Prominent recent proponents of such policies include Bernanke (2016), Buiter (2014) and Turner (2015).[3]

Other recent work has studied the implications of interest-bearing money or reserves (see, for example, Ireland, 2014). Our primary interest is in the extent to which an additional policy instrument expands the set of possible outcomes achievable by monetary and fiscal policies. We make the conventional New Keynesian assumption that the government adjusts taxes to stabilize the real present value of its nominal liabilities, for all possible paths of the price level.[4] So we abstract from potential implications of interest-bearing money operating via the government's present value budget constraint as studied by Buiter (2014) and Cochrane (2014), among others.

The rest of the paper is organized as follows. In Section 2 we set out the model. Section 3 analyzes money-financed government spending shocks, following Galí (2014a) and English et al. (2017). Section 4 examines the efficacy of money-financed transfers to households when the return on money is adjusted to ensure that households willingly hold the additional money. Section 5 investigates the sensitivity of our results to alternative assumptions about the specification of money demand. Section 6 relates our findings to several issues raised in recent discussions of the likely efficacy of monetary-financed fiscal stimulus, including the importance of allowing the rate of return on money to be adjusted. Section 7 concludes.

2 The model

This section provides a description of the baseline model, focusing on the innovations. A full derivation is presented in Appendix B.

We use an infinite-horizon model, cast in discrete time with time periods indexed by $t = 1, \ldots, \infty$. Agents in the model have perfect foresight.

We incorporate a simple financial friction that causes households to regard government liabilities as net wealth. Similarly to Castelnuovo and Nisticò (2010), Nisticò (2012) and Del Negro et al. (2015), among others, we assume that each period households face a constant probability of a default-like event that restricts participation in asset markets. Specifically, each household faces a fixed per-period probability of experiencing an 'asset reset' event that causes the household's previously accumulated assets to be lost. After a household experiences an 'asset reset' they must reformulate a new consumption plan starting from a zero asset position.[5]

[3]Many economists have used blog posts to set out the arguments for helicopter drops. See, for example, Bossone et al. (2014), Bossone (2013), Cabellero (2010), Galí (2014b), Grenville (2013), Reichlin et al. (2013), Wren-Lewis (2014) and Yates (2014).

[4]That is, fiscal policy is 'passive'.

[5]This approach is similar to the perpetual youth model of Yaari (1965) and Blanchard (1985), in which a randomly selected fraction of the population dies each period and is replaced by a cohort of newborn households. We follow the 'asset reset' interpretation because it allows the calibration of the asset reset probability to be tied to factors (other than mortality) that are likely to cause households to discount the future more heavily. Section 2.8 discusses this in more detail.

The asset reset friction effectively causes households to discount the future more heavily than otherwise. The fact that previously accumulated assets may be lost creates a disincentive to save for future consumption. Further, it implies that households will have a stronger incentive to use assets to finance current consumption, given the probability that those assets may be lost in future periods. These properties of our model imply that even temporary increases in holdings of government liabilities can have net wealth effects that stimulate consumption.

2.1 Individual households

The population consists of a continuum of households of measure one. A household that last experienced an asset reset at date j faces a budget constraint in period $t \geq j$ given by:[6]

$$\frac{M^p_{j,t}}{P_t} + \frac{B^p_{j,t}}{P_t} = \gamma^{-1}\left[\frac{R^M_t M^p_{j,t-1}}{P_t} + \frac{R_t B^p_{j,t-1}}{P_t}\right] + \tilde{w}_{j,t} - \left(1 + \varphi\left(\frac{c_{j,t}}{M^p_{j,t}/P_t}\right)\right)c_{j,t} \quad (1)$$

The household invests in money (M) and short-term government bonds (B) and receives interest income from its portfolio of money and bonds (at gross nominal rates R^M and R respectively) and real net labor income ($\tilde{w}$, defined below). Net labor income and net proceeds from portfolio changes are used to finance expenditure on consumption, c, which is measured inclusive of transactions costs (φ, discussed below). The price of consumption is denoted by P.

The rate of return on money may be positive and vary over time. Similarly to the notation of Buiter (2005), the p superscript denotes private sector demand for assets and interest rates are defined so that R_{t+1} is the (gross) rate of return on a bond held between periods t and $t+1$. So R_{t+1} and R^M_{t+1} are determined at date t.

The total returns on assets reflect the presence of the asset reset mechanism. Returns on bonds and money are adjusted by $0 < \gamma \leq 1$, which is the (constant) probability that the household reaches the following period without losing their accumulated assets. This reflects the presence of an actuarially fair insurance market that pools the risk of asset resets across households.[7]

By definition, an asset reset means that the household has no previously accumulated assets. So $M_{t-1,t} = B_{t-1,t} = 0$, for a household that experienced an asset reset in the previous period ($j = t-1$).

The real net income of the household is defined as:

$$\tilde{w}_{j,t} \equiv w_t n_{j,t} + d_{j,t} - \tau_{j,t}$$

where the household receives labor income (real wage w times labour supply, n) and lump sum transfers (either positive or negative) in the form of real dividends (d) and

[6] Our notation identifies nominal quantities and prices using upper case letters, real valued quantities and relative prices (relative to the price of consumption, P) are denoted using lower case letters.

[7] Assets taken from those households who are randomly selected for an asset reset (with probability $1-\gamma$) are redistributed proportionately among those households who do not experience an asset reset.

tax/transfer payments from the government, τ. All elements are expressed in real terms (i.e., nominal income/expenditures deflated by the price level, P).

Money provides transactions services (Schmitt-Grohé and Uribe, 2010; Del Negro and Sims, 2015). The transaction cost, φ, associated with each unit of consumption is a declining function of the household's holdings of money relative to their consumption. That is, $\varphi = \varphi\left(v_{j,t}\right)$, with $\varphi'\left(v_{j,t}\right) > 0$ and $v_{j,t} \equiv P_t c_{j,t} / M^p_{j,t}$.

We follow Del Negro and Sims (2015) and assume that the transactions cost function is given by:

$$\varphi\left(v_{j,t}\right) = Z \exp\left[-\frac{\zeta}{v_{j,t}}\right]$$

where $Z, \zeta > 0$.

The household maximizes a time-separable and additively separable lifetime utility function specified over consumption and hours worked:

$$\max \sum_{t=0}^{\infty} (\gamma\beta)^t \vartheta_t \left[\ln c_{j,t} - \frac{\chi_{j,t}}{1+\psi} n_{j,t}^{1+\psi}\right] \tag{2}$$

where ϑ_t and $\chi_{j,t}$ are exogenous shocks to utility. The first order conditions are derived in Appendix B.1.

The effective discount factor for the household has two components. The factor $0 < \beta < 1$ captures the standard assumption that households discount future utility relative to current utility. The additional factor $0 < \gamma \leq 1$ reflects the fact that previously accumulated assets are reset to zero with a probability of $1 - \gamma$ each period. Thus γ is the probability that a current consumption plan is still in effect next period.[8]

Variations in ϑ_t generate fluctuations in output and inflation that the monetary policy will seek to stabilize in the simulations studied in Section 4. The exogenous process for ϑ_t, common to all households, is:

$$\Delta \ln \vartheta_{t+1} = \rho_\vartheta \Delta \ln \vartheta_t + \varepsilon_t^\vartheta \tag{3}$$

where $\rho_\vartheta \in [0, 1)$ governs the persistence of ϑ and ε^ϑ is an exogenous disturbance.

The disutility of labor supply is also subject to a preference shifter, $\chi_{j,t}$. Although $\chi_{j,t}$ depends on the households' last reset date j as well as t, each individual household treats $\chi_{j,t}$ parametrically as it is a function of cohort-j aggregates rather than an individual household's decisions.

The first order conditions for labor and consumption derived in Appendix B.1 can be combined to give a labor supply relationship:

$$\chi_{j,t} n_{j,t}^{\psi} = \frac{w_t}{c_{j,t}\left[1 + \varphi\left(v_t\right)\left(1 + \zeta v_t^{-1}\right)\right]} \tag{4}$$

[8]The only state variables in the household's problem are asset stocks. Since these are are not carried forward in the event that the household's assets are reset, utility flows when the household experiences a reset are independent of the expected utility flow enjoyed until the point that a reset does occur. This means that the relevant maximand for the household is the latter.

where v denotes the aggregate velocity of circulation.[9]

While all households that last experienced an asset reset on the same date make identical decisions, the consumption levels of households that experienced resets at different dates will vary depending on the assets that have been accumulated in the meantime. If χ was fixed then, in principle, some households could accumulate a stock of assets to support a level of consumption sufficiently large that the solution to (4) implies an arbitrarily small number of hours worked ($n_{j,t} \to 0$), which complicates aggregation.[10]

Our choice of preferences is intended to be simple and conventional, particularly for the case in which $\gamma = 1$. However, when $\gamma < 1$, each household's consumption will increase over time, even in the steady state. If labor supply decisions depend on consumption, then a household's expected labor income and human wealth will depend on the date of their last asset reset, which complicates aggregation. To simplify, we therefore assume that $\chi_{j,t}$ evolves over time in a way that offsets the effects of cohort-specific consumption on labor supply decisions.

Specifically, we assume that

$$\chi_{j,t} = \chi \frac{c_t}{\bar{c}_{j,t}} \tag{5}$$

where $\bar{c}_{j,t}$ denotes the average consumption level of households that last experienced an asset reset in period j and c_t is aggregate consumption. Our specification of preferences therefore includes a "consumption externality in labor supply" similar to the method used by Galí et al. (2011) and Jaimovich and Rebelo (2009) to reduce wealth effects on labor supply in business cycle models. Our specification is designed not to mitigate wealth effects, but rather to remove *distributional* effects generated by differences in the date on which households last experienced an asset reset.

In equilibrium, all households that experienced a reset in the same period will make identical decisions, so that $c_{j,t} = \bar{c}_{j,t}, \forall j$. This implies that the labor supply relationship becomes

$$\chi n_{j,t}^{\psi} = \frac{w_t}{c_t \left[1 + \varphi\left(v_t\right)\left(1 + \zeta v_t^{-1}\right)\right]} \tag{6}$$

so that all households will supply the same labor, regardless of when they last experienced an asset reset. This means that the human wealth of all households is the same, facilitating aggregation.[11]

[9] Appendix B.1 shows that the form of transactions cost function generates a money demand function with the property that velocity is identical for all households.

[10] Ascari and Rankin (2007) note that for some utility functions this effect may even imply that some households wish to supply a negative quantity of hours. The authors demonstrate how a generalization of the preferences proposed by Greenwood et al. (1988) can be used to avoid the problem.

[11] In a version of the model in which there are no asset resets ($\gamma = 1$), there is a single representative household and there is no distinction between individual consumption, average cohort consumption and aggregate consumption. That is, $c_t = \bar{c}_{j,t} = c_{j,t}$. In that case $\chi_{j,t} = \chi$ and so equations (6) and (4) are equivalent.

2.2 Firms

A set of monopolistically competitive producers indexed by $j \in (0,1)$ produce differentiated products that form a Dixit-Stiglitz bundle that is purchased by households. Preferences over differentiated products are given by

$$y_t = \left[\int_0^1 y_{j,t}^{1-\eta^{-1}} dj\right]^{\frac{1}{1-\eta^{-1}}}$$

where y_j is firm j's output.

Firms produce using a constant returns production function in the single input (labor):

$$y_{j,t} = An_{j,t}$$

where A is a productivity parameter.

Firms are subject to Rotemberg (1982) price adjustment costs, so that the real profit of producer j is:

$$\frac{P_{j,t}}{P_t}y_{j,t} - w_t n_{j,t} - \frac{\Phi}{2}\left(\frac{P_{j,t}}{\pi^* P_{j,t-1}} - 1\right)^2 = \left(\frac{P_{j,t}}{P_t} - \frac{w_t}{A}\right)\left(\frac{P_{j,t}}{P_t}\right)^{-\eta} y_t - \frac{\Phi}{2}\left(\frac{P_{j,t}}{\pi^* P_{j,t-1}} - 1\right)^2$$

where $\Phi \geq 0$ is the parameter governing the strength of price adjustment costs, which are indexed to the steady-state inflation rate (the inflation target, π^*). Profits are distributed lump sum as dividends to households with each household receiving an equal share, regardless of the date at which they last experienced an asset reset.

2.3 Monetary policy

The short-term bond rate is adjusted according to a simple rule, similar to that examined by Taylor (1993), subject to an effective lower bound:

$$R_{t+1} = \max\left\{R\frac{\vartheta_t}{\vartheta_{t+1}}\left(\frac{\pi_t}{\pi^*}\right)^{\theta_\pi}\left(\frac{y_t}{y_t^f}\right)^{\theta_y}, \underline{R}\right\} \tag{7}$$

where R denotes the steady-state bond rate and $\underline{R} \geq 1$ is the effective lower bound.

Away from the effective lower bound, the policy rate is adjusted in response to deviations of inflation from the target and the output gap. The output gap is computed relative to the level of output that would prevail under flexible prices, y_t^f.[12] The coefficients $\theta_\pi > 1$ and $\theta_y > 0$ determine the strength of the policy response. The inclusion of the term $\frac{\vartheta_t}{\vartheta_{t+1}}$ in the rule incorporates an approximation to exogenous variations in the natural rate of interest that the policymaker seeks to offset.[13]

[12]As is common in models with transactions frictions, flexible price allocations are not independent of the levels of the nominal interest rates on bonds and money. We follow Kim and Subramanian (2006) and Ravenna and Walsh (2006) and define a 'supply side' flexible price equilibrium, conditional on steady-state nominal returns on money and bonds. See Appendix B.5 for a full derivation.

[13]In the absence of transactions frictions and the asset reset mechanism, the aggregate Euler equation

2.4 Fiscal policy

The period government budget constraint is:

$$M_t^g + B_t^g = R_t^M M_{t-1}^g + R_t B_{t-1}^g + P_t (g_t - \tau_t) \tag{8}$$

where the g superscript indicates that the quantities refer to government choices of asset supplies. The flow budget constraint says that the government issues money and bonds to finance its interest payments on existing liabilities and the primary deficit. The government budget constraint is written in terms of economy-wide aggregates.

We assume that the pattern of government spending is determined exogenously by:

$$\ln g_t = \rho_g \ln g_{t-1} + (1 - \rho_g) \ln g^* + \varepsilon_t^g \tag{9}$$

where $\rho_g \in [0, 1)$ controls the persistence of the process, $g^* > 0$ is the steady state level of government spending and ε_t^g is an exogenous disturbance.

As the focus of this paper is the effects of money-financed fiscal policies, for the simulations in Section 4 the government is assumed to hold the real debt stock constant:

$$b_t^g = b^*$$

where, $b_t^g = B_t^g / P_t$, though other experiments relax this assumption.

2.5 Money-financed transfers

We consider two specifications for the determination of equilibrium money holdings. In the first specification, we assume that no interest is paid on money $R_t^M = 1$. The pattern of money holdings is determined by the demand for money, given the short-term bond rate R_t set according to the policy rule (7). This is the conventional approach.

The second specification is to allow the monetary authority to directly control the stock of money M_t. This allows us to analyze the effects of a money-financed transfer to households. In this case, the stock of money is determined by a rule and the central bank adjusts R_t^M to ensure that households are willing to hold that stock. In Section 4 we use this specification to analyze the effects of money-financed transfers when the short-term bond rate is constrained by the effective lower bound.

Our baseline assumptions for fiscal policy in those experiments are that government spending and debt are held fixed in real terms. Inspection of the government budget constraint (8) indicates that, if the short-term bond rate is fixed at $\underline{R}$, an increase in M_t^g requires a reduction in nominal lump sum taxes $P_t \tau_t$. This observation leads us to interpret expansions in the money stock as money-financed net transfers, such that

would be $c_t = \frac{\pi_{t+1}}{\beta} \frac{\vartheta_t}{\vartheta_{t+1}} \frac{1}{R_{t+1}} c_{t+1}$. Under flexible prices with inflation at target, the rate of interest that keeps consumption stable is: $R_{t+1} = \frac{\pi^*}{\beta} \frac{\vartheta_t}{\vartheta_{t+1}}$. As noted previously, the presence of transactions frictions complications the definition of the flexible price equilibrium. See Appendix B.5.

the overall level of lump sum taxation falls.[14]

So a money-financed transfer is a fiscal policy action. However, since central banks generally have operational responsibility for the creation of base money, there is a debate over the feasibility of such policies under traditional institutional relationships between the central bank and fiscal authority. For example, Benigno and Nistico (2015) and Del Negro and Sims (2015) study cases in which the composition of public sector liabilities might have an effect on equilibrium allocations. These authors focus on cases in which the central bank and government have separate intertemporal budget constraints. While we believe these issues to be of practical importance, our model sidesteps this consideration by assuming that there is a single consolidated (government and central bank) budget constraint for two reasons.

First, we aim to configure the model so that there is as much chance as possible for money-financed fiscal policies to be effective. Even advocates of monetary financing in principle acknowledge the potential institutional difficulties with implementation.[15] We set these concerns aside to focus on the potential efficacy of monetary-financed fiscal policy under the assumption that such institutional difficulties can be solved.

Second, there are real-world examples of mechanisms to ensure that capital injections from the government to cover potential losses on the central bank's balance sheet arising from unconventional policies are guaranteed *ex ante*.[16] So our assumption of a single consolidated government budget constraint is not necessarily unrealistic.

2.6 Market clearing

Asset market clearing requires equality between government supply of assets and private sector demand. Our notation removes superscripts for market clearing equilibrium asset stocks.

$$b_t^p = b_t^g = b_t \tag{10}$$

$$m_t^p = m_t^g = m_t \tag{11}$$

Goods market clearing requires that output, net of adjustment costs, is purchased by the government or consumed by households:

$$y_t = c_t + g_t + \frac{\Phi}{2}\left(\frac{\pi_t}{\pi^*} - 1\right)^2 \tag{12}$$

which implies that the dividend paid by firms to each household is:

$$d_t = y_t - \frac{\Phi}{2}\left(\frac{\pi_t}{\pi^*} - 1\right)^2 - w_t n_t \tag{13}$$

[14] For a sufficiently large expansion in M_t^g, τ_t may become negative so that the government makes gross transfers to households.

[15] See, for example, Turner (2015, Chapter 14).

[16] One example of such an arrangement is the indemnity provided by the UK government on any losses sustained by the Asset Purchase Facility used by the Bank of England to conduct quantitative easing.

2.7 Aggregation

The heterogeneity across households' asset reset dates requires aggregation across these cohorts to obtain aggregate quantities. Each variable x is aggregated as follows:

$$x_t \equiv \sum_{j=-\infty}^{t} \gamma^{t-j} (1-\gamma) x_{j,t}$$

where x_t is the aggregate quantity, $x_{j,t}$ is the quantity chosen by each household that last experienced an asset reset at date $j \leq t$ and $\gamma^{t-j}(1-\gamma)$ is the share of that cohort in the population.

Appendix B.2 demonstrates that the aggregate money demand equation is:

$$m_t = \zeta^{-1} \left[\ln(\zeta Z) - \ln \frac{R_{t+1} - R_{t+1}^M}{R_{t+1}} \right] c_t \tag{14}$$

and that aggregate consumption satisfies:

$$\widetilde{c}_t = \frac{\pi_{t+1}}{\beta R_{t+1}} \frac{\widetilde{\vartheta}_t}{\widetilde{\vartheta}_{t+1}} \left[\widetilde{c}_{t+1} + (1-\gamma)\gamma^{-1} \mu_{t+1} \pi_{t+1}^{-1} \left(R_{t+1}^M m_t + R_{t+1} b_t \right) \right]$$

where μ is the marginal propensity to consume from wealth, $\widetilde{c}_t = (1+\varphi(v_t)) c_t$ denotes consumption inclusive of transactions costs and

$$\widetilde{\vartheta}_t \equiv \frac{\vartheta_t (1+\varphi(v_t))}{1+\varphi(v_t)\left(1+\zeta v_t^{-1}\right)}$$

2.8 Parameter values

Table 1 summarizes the parameter values for the baseline version of the model. Each time period is interpreted as one quarter of a year. The parameters A and χ are used to normalize steady-state output and labor supply to 1.[17] We set most parameter values to those in other studies, or to deliver the same steady-state allocations as other studies. Appendix B.6 provides details of the required calculations. Here we focus on the parameters of most relevance to our present inquiry.

We assume that the central bank's inflation target is 2% per year, consistent with the inflation targeting regimes in many economies. We choose the discount rate, β, to be consistent with a steady-state risk free real interest rate of 1.5% per year. This is somewhat lower than assumptions often used in analysis before the financial crisis. This reflects the notion that risk-free real interest rates may be somewhat lower, relative to the pre-crisis period (see, among others, King and Low, 2014; Bean, 2017; Fischer, 2016, 2017; Williams, 2017). Given our chosen value for γ (discussed below), this parameterization requires a discount factor (β) very close to unity.

[17] Appendix B.6 derives the required values to deliver these normalizations.

Table 1: Model parameters

	Value	Source/motivation
π^*	1.005	Annual inflation target of 2%
β	0.99917	Steady-state annual real interest rate $\approx 1.5\%$
γ	0.97	Del Negro et al. (2015)
g^*	0.2	Sims and Wolff (2013)
b^*	2	Reinhart et al. (2012) (advanced economies, pre-crisis)
Z	20.33	$\frac{m}{c} = 0.428$ (Del Negro and Sims, 2015)
ζ	25.75	Del Negro and Sims (2015)
η	7.88	Rotemberg and Woodford (1997)
Φ	80.83	Calvo (1983) price adjustment probability ≈ 0.25
ψ	0.55	Smets and Wouters (2007)
θ_π	1.5	Taylor (1993)
θ_y	0.125	Taylor (1993)
$\underline{R}$	1.0006	Effective lower bound of 25bp (annualised)

We set the parameters governing money demand (Z and ζ) to deliver the same steady-state velocity and elasticity of real money balances with respect to the nominal interest rate estimated by Del Negro and Sims (2015) using US data.[18]

The most important parameter for generating net wealth effects is $1-\gamma$, the probability that a household transitions to a state in which it has no assets. Del Negro et al. (2015) calibrate γ with reference to the probability of events that lead to transition to a state of default or other constraints on using assets to finance spending. These considerations lead them to set $\gamma = 0.97$, which we adopt as our baseline value.

There is ample empirical evidence that households discount the future even more heavily than implied by our baseline calibration. Experimental evidence generates a wide range of estimates. The averages of the lower and upper bound estimates surveyed by Frederick et al. (2002) suggests values for γ of 0.86 and 0.95 respectively. The posterior mean estimated by Castelnuovo and Nisticò (2010) using macroeconomic data implies $\gamma = 0.87$.

2.9 Simulation approach

The simulations in Sections 3 and 4 are consistent with the perfect foresight assumption under which the model is derived. The perfect foresight assumption allows us to consider non-linear effects (in particular of money demand when the return on money approaches that on bonds) without requiring the use of projection methods to solve the model. In our simulations, we assume that in period $t = 0$ the economy is at its deterministic steady state. At the beginning of period $t = 1$ information about the exogenous disturbances and the behaviour of policy is revealed. In particular, announced temporary policies (such as an expansion of the monetary base) are regarded as fully credible by private agents. Perfect foresight implies that equilibrium

[18] The value for ζ is one quarter of the value reported by Del Negro and Sims (2015) because their estimation uses annualized interest rates.

outcomes are consistent with the information revealed at the beginning of period $t = 1$. The TROLL modeling software is used to compute the equilibrium outcomes.

3 Pitfalls of money-financed government spending

In this section we consider experiments in which monetary policy is specified such that changes in government spending are financed by money creation.

3.1 Stimulus from money-financed government spending

We first consider the effect of financing a government spending increase by money creation (rather than debt issuance) in a similar manner to Galí (2014a). We consider a temporary exogenous increase in government spending. Government spending is determined by (9) and we set $\varepsilon_1^g = 0.05$ and $\varepsilon_t^g = 0, t = 2, \ldots$. The persistence parameter is set to $\rho_g = 0.9$ which corresponds to the "high persistence" calibration used by Galí (2014a). We choose this calibration because it is associated with large effects of money-financed government spending increases in Galí's model.

We examine the macroeconomic effects for two alternative assumptions about the conduct of monetary and fiscal policy. In both cases, we adopt the conventional assumption that money earns no interest (so that $R_t^M = 1, \forall t$).

In the first case ('debt financing'), higher government spending is financed by issuing short-term debt. A fiscal rule adjusts the lump sum tax to ensure that real government debt returns to b^* in the long run. However, this adjustment is not immediate so that, in the short run, government debt rises as the government borrows to finance the additional spending. We assume that the following fiscal rule for the lump sum tax acts to stabilize the value of government liabilities in the long run:

$$\tau_t = \tau^* + \theta_b \left(b_{t-1} - b^* \right) \tag{15}$$

where $\theta_b > 0$ determines the strength of the fiscal feedback and τ^* is the steady-state value of the tax. However, this rule is only activated $K + 1$ periods ($K \geq 0$) after the initial increase in government expenditure, so that $\tau_t = \tau^*, t = 1, \ldots, K$. In our simulations, we set $K = 12$, which mimics Galí's analysis with fixed tax rates.[19] The short-term interest rate, R_t, is determined by the Taylor rule, (7).

The second case ('money financing') assumes that government debt is held constant: $b_t = b^*, \forall t$. We also assume that lump sum taxes are held constant: $\tau_t = \tau^*, \forall t$. For the

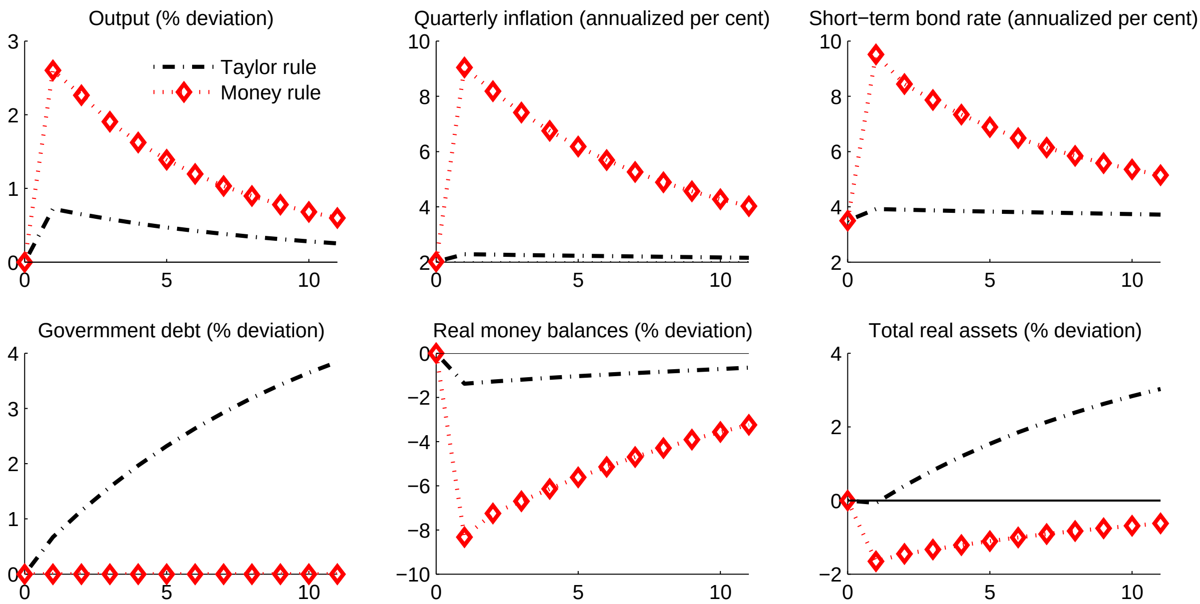
[19]Specifically, the fiscal rule is assumed to be inactive for K periods (during which time $\tau_t = \tau^*$) and from period $K + 1$ taxes are set according to (15) with $\theta_b = 0.1$. Galí (2014a) abstracts from changes in tax rates. Since he uses a model with $\gamma = 1$, it is legitimate to assume that taxes are held fixed for an arbitrary period, as long as there is an eventual adjustment in taxes to ensure that the government's solvency condition is satisfied. The nature of Galí's model means that equilibrium allocations are invariant to the pattern of taxes as long as government liabilities are eventually stabilized. Our model features net wealth effects, so in principle the horizon K over which taxes are held fixed will matter. Quantitatively, however, results are almost identical to those shown here for choices of $K > 12$. Appendix A.1 shows results for the case in which $\gamma = 1$, which replicates Galí's set up.

government's budget constraint to hold, money balances must be adjusted according to:

$$m_t = R_t^M \pi_t^{-1} m_{t-1} + \left(R_t \pi_t^{-1} - 1 \right) b^* + g_t - \tau^* \tag{16}$$

Because both taxes and government debt are held fixed in this case, real money balances must be adjusted to satisfy the government budget constraint. To ensure that households willingly hold the required level of real money balances, the interest rate on short-term government debt must adjust. So the money creation rule (16) is used *in place of* the Taylor rule (7). Thus Galí's experiment amounts to considering a government spending shock under alternative policy rules that determine the short-term bond rate, R_t, as well as those that determine debt and taxes.

Figure 1: A government spending increase under debt financing and money financing



Notes: The model starts in period 0 at the steady state. Government spending is given by (9). The panels show the responses to $\varepsilon_1^g = 0.05$ (with $\varepsilon_t^g = 0, t \geq 2$). In the Taylor rule case, the tax rate fiscal rule is initially inactive so that $\tau_t = \tau^*$ for $t = 1, \ldots, K$, where $K = 24$. From period $t = K + 1$ taxes are set according to (15) with $\theta_b = 0.1$. The short-term interest rate is set according to (7). In the money rule case, the tax rate and debt stock are held constant and the government spending increase is financed by money creation according to equation (16). There is no interest paid on money ($R_t^M = 1, \forall t$) so the short term interest rate R_t adjusts to ensure that (16) holds.

Figure 1 shows the results. We observe that an expansionary government spending shock generates much more stimulus under 'money financing' than under 'debt financing', replicating Galí's results. However, Figure 1 also reveals that the additional stimulus generated by money financing is not associated with stronger wealth effects (via higher real asset values). Indeed, real assets fall in this case (bottom right panel). Instead, money financing is associated with a lower path for real interest rates which stimulates spending through the Euler equation for consumption.

To see this, recall that consumption satisfies:

$$\widetilde{c}_t = \frac{\pi_{t+1}}{\beta R_{t+1}} \frac{\widetilde{\vartheta}_t}{\widetilde{\vartheta}_{t+1}} \left[\widetilde{c}_{t+1} + (1 - \gamma) \gamma^{-1} \mu_{t+1} \pi_{t+1}^{-1} a_{t+1} \right]$$

where $a_{t+1} \equiv R^M_{t+1} m_t + R_{t+1} b_t$ represents real financial assets.

This means that current consumption can be increased by reductions in the real interest rate $\frac{R_{t+1}}{\pi_{t+1}}$ and increases in the real value of assets a_{t+1}. As noted above, real asset values fall in response to the government spending increase under money financing. However, the real interest rate also falls: inflation rises materially, but the short-term bond rate increases by (slightly) less.[20]

To confirm this intuition Appendix A.1 repeats this experiment for a version of the model with $\gamma = 1$, so there are no net wealth effects. The results in Figure A.1 are virtually indistinguishable from those in Figure 1. This underscores the fact that the key mechanism at work is the real interest rate channel, since the consumption equation collapses to $\widetilde{c}_t = \frac{\pi_{t+1}}{\beta R_{t+1}} \frac{\widetilde{\vartheta}_t}{\widetilde{\vartheta}_{t+1}} \widetilde{c}_{t+1}$ when $\gamma = 1$.

In a recent paper, English et al. (2017) also replicate Galí's result and note the importance of the extent to which monetary policy accommodates the inflationary impetus of the government spending increase. Importantly, they demonstrate that the money-financing rule (a variant of (16)) can be represented as a rule for the short-term bond rate (R) that responds to the deviation of the price level from a target path. The target path is determined by the level of government spending such that small increases in spending generate a large rise in the target. As a result, monetary policy accommodates a temporary but substantial rise in inflation so that the price level moves up to the new target path.

The analysis of English et al. (2017) uses the well-known result that in standard models (with no interest on money) there is an equivalence between a policy rule written in terms of the money stock (such as (16)) and a policy rule written in terms of the short-term bond rate, R. Indeed, Eggertsson and Woodford (2003, p147) use this result to develop "an irrelevance proposition for open-market operations in a variety of types of assets that the central bank might acquire, under the assumption that the open-market operations do not change the expected future conduct of monetary or fiscal policy". Their result implies that any macroeconomic effects that can be generated by a particular type of policy rule specified in terms of the money stock can also be achieved by an appropriately specified interest rate rule. From this perspective, Galí's policy prescription can be viewed as advocacy of a particular rule for the *short-term bond rate*, the form of which is uncovered and analyzed by English et al. (2017).

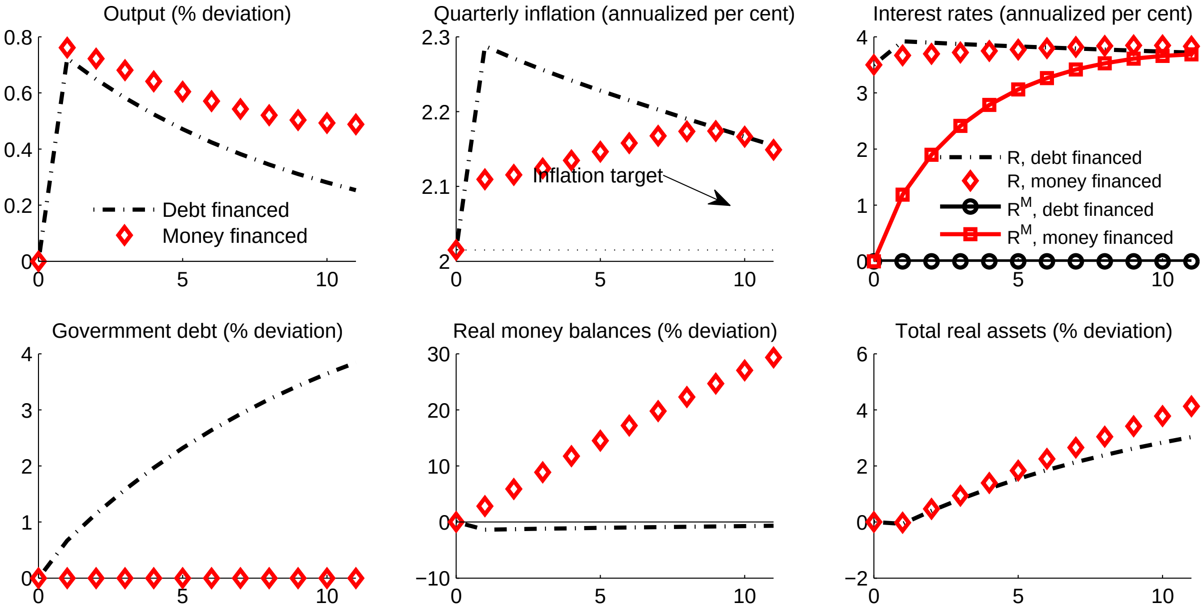
3.2 Financing government spending with interest-bearing money

To demonstrate the importance of policy behaviour in determining the effects of money-financed government spending, we now examine a case in which the short-term bond rate continues to be determined by the Taylor rule (7). To implement this variant, real money balances must therefore satisfy (16) while the short-term bond rate satisfies (7). To achieve this, the interest rate on money, R^M, is adjusted to ensure that

[20] In Figure 1 inflation rises by around 7 percentage points and the short-term bond rate by around 6 percentage points (both measured relative to steady state).

the additional money created to finance the additional government spending increase is willingly held.

Figure 2: Government spending increase financed via interest-bearing money vs debt



Notes: The model starts in period 0 at the steady state. Government spending is given by (9). The panels show the responses to $\varepsilon_1^g = 0.05$ (with $\varepsilon_t^g = 0, t = 2, \ldots$). In the debt financed case, the tax rate fiscal rule is initially inactive so that $\tau_t = \tau^*$ for $t = 1, \ldots, K$, where $K = 12$. From period $t = K + 1$ taxes are set according to (15) with $\theta_b = 0.1$. In the money financed case, the tax rate and debt stock are held constant and the government spending increase is financed by money creation according to equation (16). The return on money is adjusted to ensure that households willingly hold the additional money balances.

Figure 2 demonstrates that, when the return on money (R^M) adjusts, the effects of the government spending increase are much more similar, regardless of whether it is financed through debt or money creation. The bottom row reveals that the two financing arrangements have markedly different implications for the paths of government debt and real money balances. However, the path for total real assets (a_t), which is the relevant determinant of consumption expenditure in the model, is very similar for the two financing approaches.

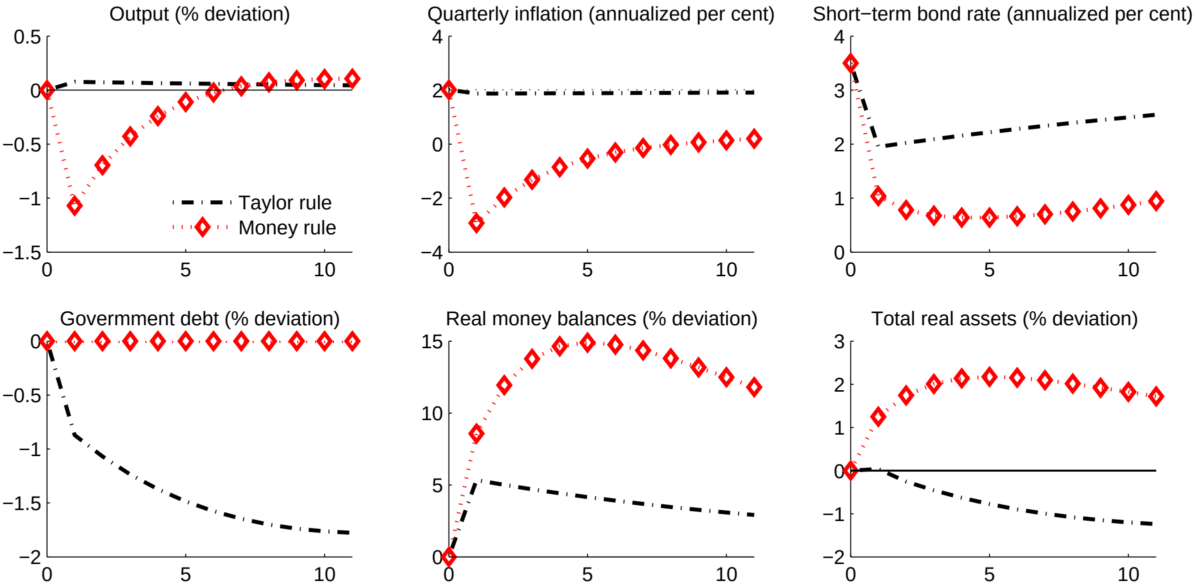
A key reason for the difference between the responses for debt-financed and money-financed government spending increases in Figure 2 is that the consumption Euler equation is 'tilted' by the change in the transactions costs associated with holding money. Appendix A.2 demonstrates that in a variant of the model in which the demand for money is additively separable, the responses from money and debt financed government spending are almost identical.

These results show that the effects of a money-financed government spending increase depend on the precise monetary policy arrangements in place. If the short-term bond rate R is set using a rule with a strong response to inflation (as in (7)), then the inflationary effects of the government spending increase are contained (as in Figure 2). This result is only achievable if the rate of return on money is allowed to rise, so that the additional money created to finance the spending increase is willingly held. If no interest is payable on money, then the Taylor rule (7) must be abandoned to allow the short-term bond rate R to adjust to deliver the monetary financing rule (16). In that case, the implicit interest rate feedback rule has a weak response to inflation (as in Figure 1).

3.3 Broader implications of a weak policy response to inflation

The preceding results indicate that the ability of money-financed government spending increases to stimulate demand and inflation stems from the fact that such policies imply a weak response of the short-term bond rate to inflationary developments. Indeed, part of the rationale for such policies is that such a response may be beneficial in some circumstances: if the economy is stuck at the effective lower bound, then pursuing a policy that increases inflation expectations (and ultimately inflation) may be very desirable. However, replacing the Taylor rule (7) with the money financing rule (16) will affect the equilibrium responses to *all* shocks, not just those that provide stimulus such as an increase in government spending.

Figure 3: Responses to a preference shock under alternative monetary policy rules



Notes: The panels show responses to an unanticipated preference shock ε^{ϑ} in period 1. In the 'Taylor rule' case, taxes are set according to (15) with $\theta_b = 0.1$. The short-term bond rate is set according to (7). In the 'money rule' case, the tax rate and debt stock are held constant and the government budget constraint is enforced by money creation according to equation (16). There is no interest paid on money ($R_t^M = 1, \forall t$) so the short-term bond rate R_t adjusts to ensure that (16) holds.

For example, Figure 3 compares the responses to a preference shock, ε^{ϑ}, when the short-term rate R is determined by the money rule (16) (red lines with diamond markers) and the Taylor rule (7) (black dash-dot lines).[21] Under the Taylor rule, the short-term policy rate is cut sharply to offset the variation in the natural real interest rate generated by the preference shock. Inflation stays close to target and output is

[21] In both cases, the model starts in steady state in period 0. The preference shock is determined by equation (3), with $\rho_{\vartheta} = 0.85$. In period 1, the preference shock is $\varepsilon_1^{\vartheta} = 0.005$ (with $\varepsilon_t^{\vartheta} = 0, t = 2, \ldots$).

barely changed.[22] In contrast, under the money rule, the short-term bond rate rate is determined by (16), which ensures that the debt stock is held fixed by adjusting the rate of money growth. In equilibrium, this generates a a rise in the real interest rate, leading to a recession and prolonged undershoot of inflation. This example illustrates the far-reaching implications of *permanently* replacing a standard Taylor-type monetary policy rule with a monetary financing rule.

4 Money-financed transfers at the effective lower bound

While many recent policy proposals have focused on money-financed government spending increases, similar to those investigated in the previous section, Friedman's original thought experiment was cast as a direct monetary transfer to households. Whether or not such a transfer stimulates spending depends on households' reaction to an increase in nominal income and the extent to which the transfer is permanent. The original 'helicopter drop' experiment assumed a one-off, permanent increase in the supply of money. In this section we explore the consequences of experiments in which the money stock is increased with varying degrees of permanence.

Importantly, we assume that the rate of return on money is adjusted to ensure that the monetary injection is willingly held by households and that the short-term bond rate is determined by the Taylor rule (7). The objective is to explore whether it is possible to achieve a stimulative effect from expanding the stock of money without requiring a *permanently* weak response of the short-term bond rate to inflation. As we saw in the previous section, such a weak response to inflation may be beneficial in some circumstances, but not others.

4.1 A recessionary scenario

To explore the potential for money-financed transfers to stimulate the economy, we build a simulation in which the short-term bond rate hits the zero bound. As in previous sections we assume that a shock arrives in period $t = 1$, with the model at steady state in period $t = 0$. Specifically, we consider a case in which a large preference shock (ϑ) generates a recession large enough to constrain the monetary policy rule (7) at the effective lower bound $\underline{R}$. We assume that $\varepsilon_1^{\vartheta} = 0.0115$ with $\varepsilon_t^{\vartheta} = 0, t = 2, \ldots$. The preference disturbance evolves according to equation (3), with ρ_{ϑ} set to 0.95 to generate a persistent spell at the effective lower bound.

We assume that the effective lower bound on the nominal interest rate is 0.25%. This is broadly consistent with the experience of some advanced economies (eg the United Kingdom and United States) but not for those economies that have implemented negative policy rates (including the Euro area and Japan). We require the effective lower bound to be strictly positive, otherwise the demand for money (in the baseline simulation) would be infinite. From a practical perspective, the fact that

[22]Full stabilization is not achieved because the measure of flexible price output in the policy rule is computed conditional on the assumption that the short-term bond rate is fixed.

large economies have successfully implemented negative policy interest rates without explosive increases in the demand for money suggests that the true rate of return on money is likely to be slightly negative (for example, reflecting storage and security costs that do not appear in our model).[23]

Our baseline assumption for policy is that the return on money is fixed at $R_t^M = 1, \forall t$ and that the short-term bond rate R evolves according to the monetary policy rule (7).

4.2 Money-financed transfers with interest-bearing money

We now consider what happens if a temporary money-financed transfer to households is announced in period $t = 1$. The temporary transfer is determined by the following process for the aggregate nominal money stock:

$$\frac{M_t}{M_{t-1}} = \left(\frac{M_{t-1}}{M_{t-2}}\right)^{\rho_m} (\pi^*)^{(1-\rho_m)} \exp\left(\varepsilon_t^m\right) \tag{17}$$

for periods $t = 1, \ldots, K$. For the duration of the monetary expansion, money growth follows an autoregressive process, which we assume to be weakly persistent by setting $\rho_m = 0.33$. The process is driven by a disturbance term ε^m and we set $\varepsilon_1^m = 1.01$, with $\varepsilon_t^m = 0$ for $t = 2, \ldots, K$. We consider a two year monetary expansion ($K = 8$). For periods $t = 1, \ldots, K$, we assume that the policymaker adjusts the rate of return on money (R_t^M) to ensure that the additional money stock is willingly held by households.

Our assumption of a temporary monetary expansion is intended to facilitate comparisons with variants of the model that contain alternative monetary frictions (analyzed in Section 5). Moreover, a temporary monetary expansion during a period in which the short-term bond rate is constrained by the effective lower bound may be sufficient to stimulate spending and inflation by ensuring that the policy response to higher inflation expectations is *temporarily* weaker than usual. We consider a more permanent monetary expansion in Section 4.3.

We calibrate the size of the monetary injection with reference to the quantitative easing experiences of the United Kingdom and United States following the financial crisis. While we emphasize that we are *not* analyzing the effects of quantitative easing, specifying the transfer with reference to the scale of money expansion associated with those policies is intended to ensure that we are considering a policy intervention that is 'large', but not unprecedented in recent economic history.

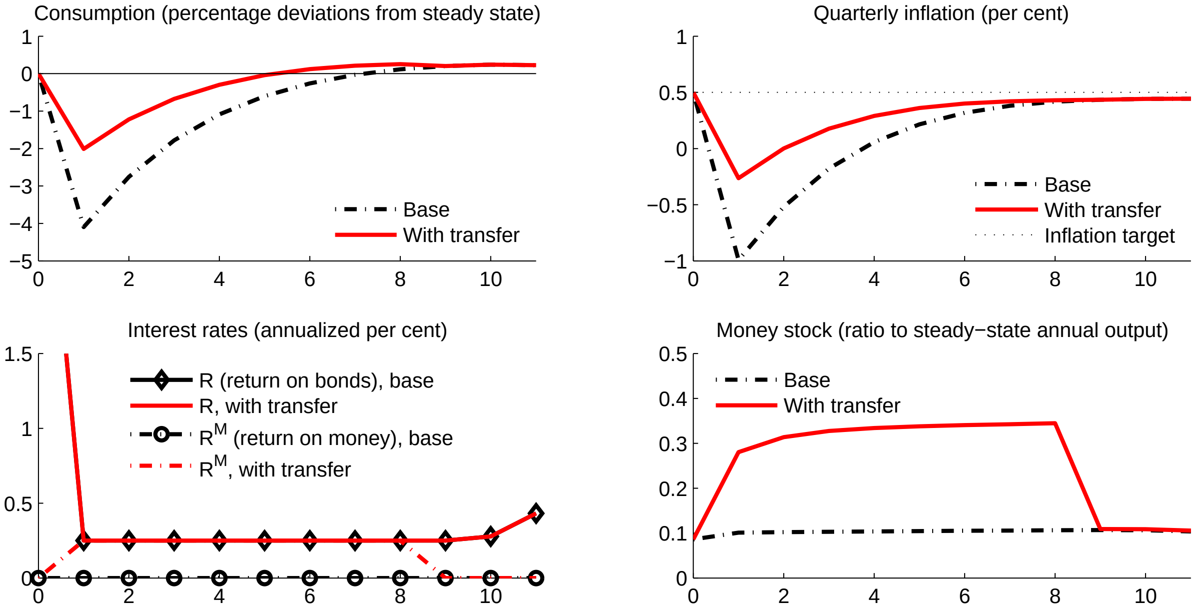
Reis (2016) documents the evolution of the balance sheets of major central banks between 2007 and 2015. If we interpret 'money' in our model as a composite of currency and interest bearing reserves, then Reis (2016, Figure 1) suggests that the stock of money increased by around 25 percentage points of (annual) GDP between 2007 and 2015 in both the United States and United Kingdom. For our simulation, the monetary injection is measured relative to *steady-state* GDP and calibrated to be

[23] Alternatively, there may be a zero bound on deposit rates (again abstracted from in our model) that does not apply to the overnight policy rate.

approximately 25pp.[24]

Figure 4 shows the results. The black (dash-dot) lines show the baseline simulation, without a monetary transfer. The short-term bond rate is immediately cut from its steady-state level of 3.5% to the lower bound of 0.25% and remains there for nine quarters. Given the scale of the shock, however, this policy response is insufficient to stabilise spending and inflation. Consumption falls by 4% in period 1 and *quarterly* inflation undershoots the target by 1.5 percentage points.

Figure 4: A money-financed transfer at the effective lower bound



Notes: The model starts in period 0 at the steady state and a recessionary shock $\varepsilon_1^{\vartheta} = 0.0115$ arrives in period 1. The black dashed lines show the effects of the recessionary shock. The solid red lines show the case in which a temporary money-financed transfer is used to combat the effects of the recessionary shock. For periods $t = 1, \ldots, 8$, the money stock is determined by (17). The value of ε_1^m is chosen to deliver the desired total increase in the money stock and $\varepsilon_t^m = 0$ for $t = 2, \ldots, 8$. For the duration of the money-financed transfer, the rate of return on money R^M is endogenously determined. From period 9 onwards, the rate of return on money is fixed at unity and the quantity of money is determined by households' demand for money.

The solid red lines in Figure 4 show the effect of the shock when a money-financed transfer is also announced at the start of period 1. The transfer ends in period 8, so that from period 9 onwards no interest is paid on money and the level of real money balances is determined by household demand. This would be akin to central banks that did not pay interest on reserves prior to the crisis temporarily paying interest on reserves, before reverting to the pre-crisis policy as the economy recovers.

The expansion in the money stock requires an increase in the return on money (bottom left panel, dashed red line) so that households are willing to hold the additional money balances. The monetary transfer increases consumption by around 2 percentage points in period 1. Quarterly inflation is around 0.8 percentage points higher in period 1. The monetary transfer stimulates the economy via a direct wealth

[24]Reis reports stocks of assets and liabilities as a proportion of actual GDP. We abstract from the change in GDP over the period in question because it is small relative to the observed changes in reserves and currency.

effect as the real money balances held by households increase. Because there is no conventional monetary policy response to the stimulus (the short-term bond rate does not change relative to the baseline simulation), inflation expectations increase and the expected real interest rate falls. This provides a further boost to consumption.

As discussed in Section 2.5, an expansion of M at the lower bound leads to a reduction in taxes $P\tau$, given our assumptions about government spending and debt. Because the expansion in M is temporary, there is a sharp fall in M when the transfer ends, corresponding to a *rise* in $P\tau$. In the absence of the asset reset mechanism, households would save the additional income from the temporary transfer, to pay the subsequent increase in taxes. However, in the presence of the asset reset mechanism there is a risk that households will experience an asset reset before the tax rise occurs. In that case, they will experience the increase in taxes without having additional assets from which to finance it. This creates an incentive to spend some of the additional income generated by the money-financed transfer.

We have seen that when the short-term bond rate is temporarily constrained at the lower bound, it does not rise to counteract the stimulative effects of the money-financed transfer. As in the experiments of Section 3.1, a policy intervention that increases inflation has more effect when the short-term bond rate responds weakly. However, in this case the lack of a short-term bond rate response is a *temporary* consequence of the lower bound constraint rather than a permanent change in the monetary policy rule. As discussed above, one of the main reasons for advocating money-financed policies is to provide stimulus when other monetary policy instruments are constrained.

4.3 A 'permanent' money-financed transfer

It is generally argued that monetary transfers that are permanent (as in Friedman's original thought experiment) are more effective than temporary transfers. Indeed, some authors argue that achieving *any* stimulus via monetary transfers at the zero bound requires those transfers to be permanent (Krugman, 1998; Eggertsson and Woodford, 2003).

The monetary transfer in Section 4.2 lasts for just eight quarters. After the policy intervention, the interest rate on money returns to zero and the monetary policymaker supplies whatever quantity of money is demanded by households at this rate. As described in Section 4.2, there is a *withdrawal* of money from households (requiring higher taxes) when the policy ends.

The fact that a temporary transfer stimulates spending reflects the fact that it operates via a wealth channel and can be implemented without changing the path of the short-term bond rate (because the interest rate on money is adjusted appropriately).[25] Nevertheless, it is instructive to consider whether a more permanent money-financed transfer would be more powerful.

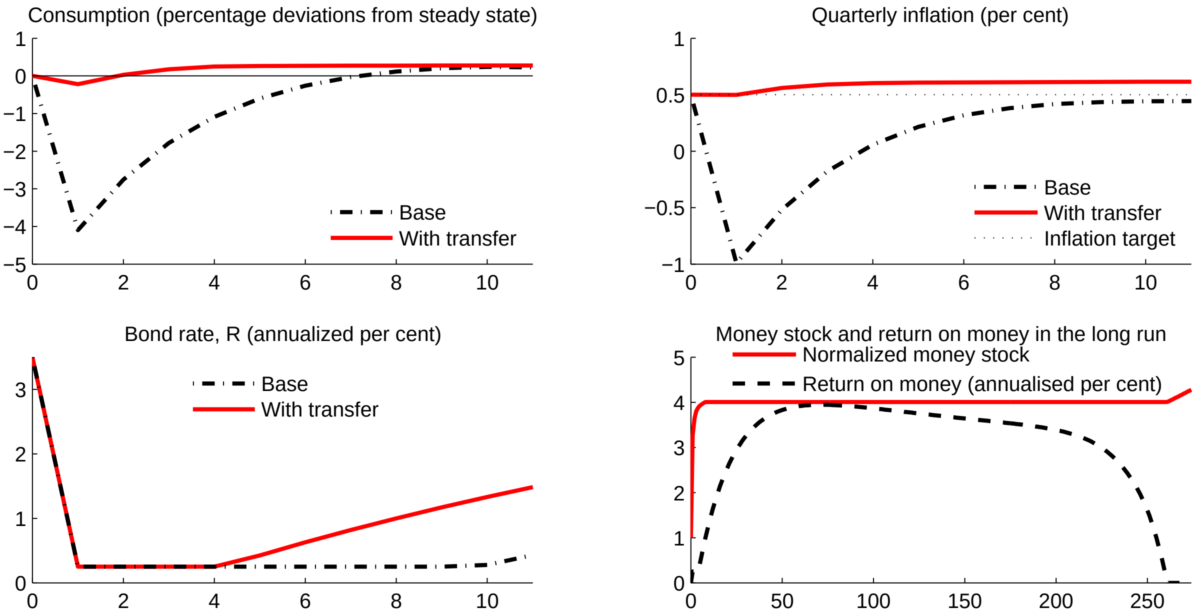
[25]The results of Krugman (1998) and Eggertsson and Woodford (2003) rely on the duality between the operation of a policy in terms of a path for the short-term bond rate and the quantity of money. A policy that delivers a permanently higher stock of money must also deliver a permanently higher price level. In forward-looking models that is achieved by a temporarily higher inflation rate generated by a path for the short-term bond rate that responds weakly (if at all) to higher inflation.

In this section, we consider an experiment in which the policymaker never withdraws the stock of money initially transferred to households. To implement this experiment we assume that the nominal money stock is determined by:

$$M_t = \begin{cases} M_{t-1}\left(\frac{M_{t-1}}{M_{t-2}}\right)^{\rho_m}(\pi^*)^{(1-\rho_m)}\exp\left(\varepsilon_t^m\right) & t = 1,\ldots,K \\ \max\left\{M_{t-1}\,,\,\zeta^{-1}\left[\ln\left(\zeta Z\right) - \ln\frac{R_{t+1}-1}{R_{t+1}}\right]P_t c_t\right\} & t = K+1,\ldots \end{cases} \tag{18}$$

We set $K = 8$ and use the same value of ε_1^m as in Section 4.2. These assumptions mean that the monetary expansion follows the same path as the experiment in Section 4.2 for $t = 1,\ldots,K$. Equation (18) specifies that the money stock is held constant for periods $t = K+1,\ldots$ unless that value exceeds $\zeta^{-1}\left[\ln\left(\zeta Z\right) - \ln\frac{R_{t+1}-1}{R_{t+1}}\right]P_t c_t$. Inspection of (14) reveals that this quantity corresponds to the demand for *nominal* money when the net interest rate on money is zero ($R_{t+1}^M = 1$). So this specification requires that the initial increase in the money stock is maintained as long as the rate of return required for the money stock to be willingly held is non-negative.

Figure 5: A permanent money-financed transfer at the effective lower bound



Notes: The model starts in period 0 at the steady state and a recessionary shock $\varepsilon_1^\vartheta = 0.0115$ arrives in period 1. The black dashed lines show the effects of the recessionary shock. The solid red lines show the case in which a permanent money-financed transfer is used to combat the effects of the recessionary shock. In this case the money stock is determined by (18). Responses in the bottom right panel are plotted for 275 quarters. The nominal money stock is normalized to 1 in period 0.

Figure 5 shows the results of this experiment. Comparing the top row with the top row of Figure 4 reveals that the permanent money-financed transfer has a much larger macroeconomic effect than a temporary transfer. Other things equal, the permanent transfer has a larger net wealth effect and hence a more expansionary effect on aggregate demand and inflation. Indeed, the permanent transfer is sufficiently stimulative that it brings forward the date at which the short-term bond rate lifts off from the zero bound.

The bottom right panel of Figure 5 (showing the *long-run* implications of the transfer) shows that the permanent monetary transfer requires a strictly positive rate of return on money for more than 65 years. The 'hump shaped' response on the return on money reflects the strength of the wealth effects in the near term. This drives up the short-term bond rate (which rises to offset the wealth effects on consumption and hence output) and hence the return on money required to induce households to hold the additional money balances. In the longer term, the price level rises in line with the inflation target and the return on money required for households to hold a given nominal stock of money falls.

Much of the power of the permanent money-financed transfer is in fact driven by the nature of the reaction function for the short-term bond rate, R. As described in Section 2.3, the reaction function responds to a measure of the output gap based on a 'supply side' concept of flexible price equilibrium. As a result, the policy rule (7) does not fully stabilize spending and inflation away from the lower bound. Indeed, the money-financed transfer is sufficient in this case to generate a small, but persistent, overshooting of the inflation target. Section 5.2 considers a variant of the model in which the reaction function for the short-term bond rate *does* completely stabilize spending and inflation away from the lower bound. In that case there is a limit to the extent to which a permanent money-financed transfer can provide additional stimulus.

Finally, we note that a bond-financed transfer of a similar magnitude could also generate similar stimulative effects on consumption and inflation at the zero bound. The primary friction through which the stimulus operates is the asset reset friction, which applies to all government liabilities.

5 Robustness to alternative monetary frictions

In this section we consider the robustness of our results to alternative assumptions about the underlying frictions that give rise to the demand for money.

5.1 A cash in advance friction

In Appendix C we develop a variant of the model in which the demand for money arises from a cash in advance constraint. This approach greatly reduces the sensitivity of money demand to the interest differential between money and bonds. On the other hand, a simple cash in advance assumption typically implies much larger average money holdings than observed in the data.[26]

The cash in advance variant assumes that consumption spending must be financed by existing money holdings brought forward from the previous period, income from maturing bonds (net of new bond purchases) and a transfer from the government. Money is assumed to earn no interest.

[26] One modification to address this is to assume that only a subset of consumption goods are subject to the cash in advance constraint with the remainder being 'credit goods'.

The cash in advance constraint will bind if the rate of return on bonds is strictly positive: households will hold only the money balances required to finance consumption, allocating the remainder of their portfolio to bonds. However, when the return on bonds is zero the cash in advance constraint will not bind and households are indifferent between allocating their portfolios between money and bonds. This means that an expansion in the stock of money beyond the level required to finance consumption expenditures is willingly held.

These assumptions give rise to a model with almost identical behavioural equations to the baseline variant. The key differences are in the consumption equation and the Phillips curve, which Appendix C shows to be:

$$c_t = (1-\mu_t)^{-1}\left[\frac{\gamma\pi_{t+1}}{R_{t+1}}\frac{\mu_t}{\mu_{t+1}}c_{t+1} + (1-\gamma)\,R_{t+1}^{-1}\mu_t\left(R_{t+1}b^* + m_t\right)\right]$$

$$\frac{\Phi\pi_t}{y_t\pi^*}\left(\frac{\pi_t}{\pi^*}-1\right) = 1-\eta+\eta\chi y_t^{\psi}c_t R_{t+1} + \frac{\Phi}{y_t}\frac{\pi_{t+1}}{R_{t+1}}\frac{\pi_{t+1}}{\pi^*}\left(\frac{\pi_{t+1}}{\pi^*}-1\right)$$

The consumption equation, while very similar to the baseline model, contains a slightly different term that gives rise to the wealth effect from government liabilities.[27] However, it is also clear that the wealth effects disappear when $\gamma = 1$, as in the baseline version of the model. The Phillips curve equation has the feature that marginal cost is an increasing function of the interest rate on short-term bonds: there is a 'cost channel' (Barth and Ramey, 2002). This arises because the marginal rate of substitution between consumption and leisure depends on the Lagrange multiplier on the cash in advance constraint and (hence) the nominal interest rate.

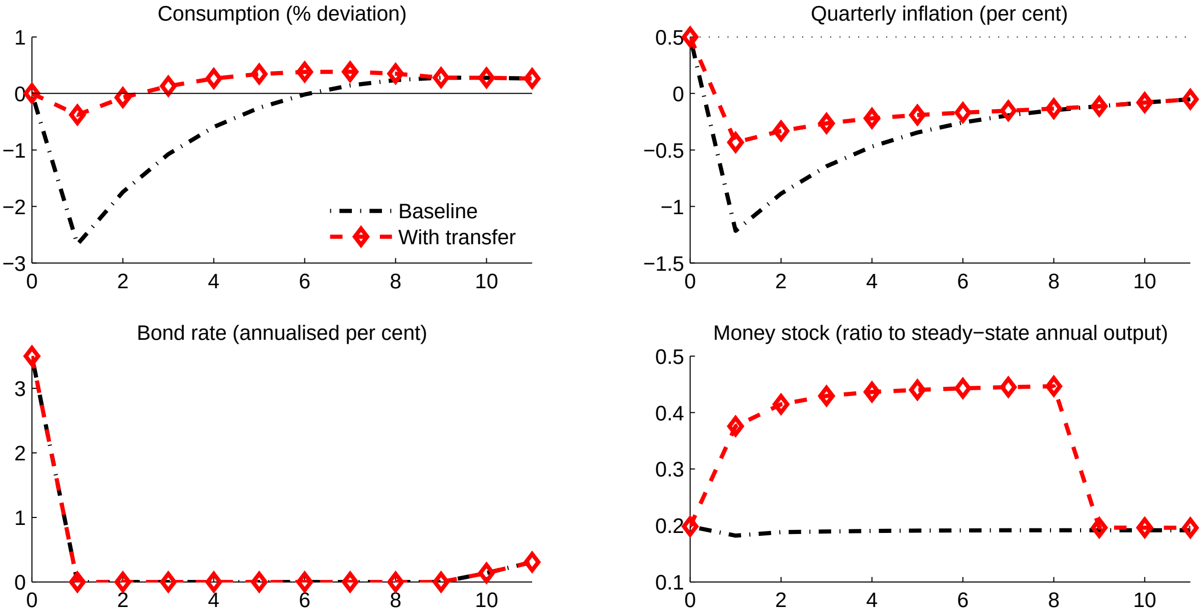
Figure 6 shows the results of an experiment similar to that considered in Figure 4 using the cash in advance variant of our model. Specifically, the model begins in period $t = 0$ in steady state. In period $t = 1$, a shock to ϑ generates a fall in demand, prompting a cut in the interest rate on short-term bonds via the Taylor rule. The shock is sufficiently large that the short-term interest rate is constrained by the effective lower bound. In this variant of the model, we set the lower bound on the net short-term bond rate to zero ($\underline{R} = 1$).

When the lower bound is binding, the cash in advance constraint will be slack and households will be willing to hold money balances in excess of the minimum quantity required for consumption purposes. Our baseline assumption in Figure 6 is that the government monetary transfer is adjusted to ensure that households hold the minimum quantity of money required to financed consumption expenditure. This assumption is intended to be a neutral benchmark against which to assess alternative policy options.

The baseline recessionary scenario is plotted as black dash-dotted lines in Figure 6. As in the experiment in Section 4.2, the shock generates a substantial fall in consumption and inflation and the interest rate on short-term bonds remains at the

[27]This difference implies that a slightly different calibration for β is required to deliver the same steady state return on bonds as the baseline model. All other parameter values in the simulations below are unchanged from the baseline model.

Figure 6: A money-financed transfer at the effective lower bound: cash in advance model



Notes: The model starts in period 0 at the steady state and a recessionary shock $\varepsilon_1^{\vartheta} = 0.005$ arrives in period 1, which drives the economy to the zero lower bound in the baseline simulation (black dash-dot lines) for 9 quarters. In the baseline simulation, while the economy is at the zero bound the money stock is adjusted to the level at which the cash in advance constraint *would* (just) bind if operative. In the case of a money-financed transfer (red dashed lines with diamond markers), for periods $t = 1, \ldots, 8$ the path of the money stock delivers a 25pp increase in nominal money balances relative to steady-state annual GDP. From period 9 onwards, the money stock is determined by the cash in advance constraint, which binds.

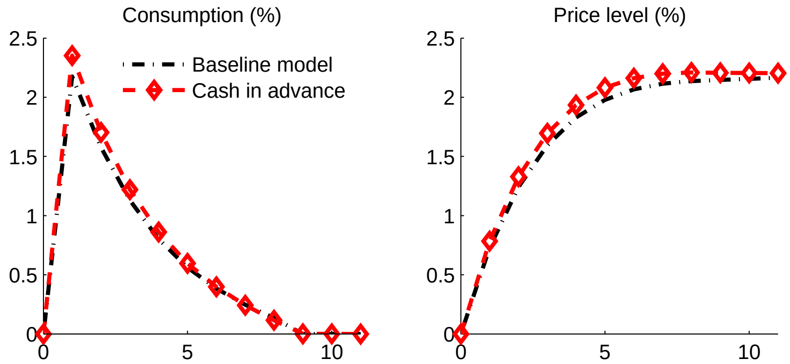
zero bound for ten quarters before rising very gradually. The recession is associated with a small reduction in the money stock. That reflects the fact that the level of money balances required to satisfy the cash in advance constraint responds more strongly to the level of economic activity than to the nominal return on short-term bonds.

The dashed red lines with diamond markers in Figure 6 depict the effect of a temporary monetary-financed transfer to households. This monetary expansion is calibrated to deliver the same increase in nominal money balances as the experiment in Section 4.2, which implies a smaller proportionate increase in money balances given the larger steady state money stock in this variant of the model. The results indicate that large monetary expansions can be effective at the zero lower bound even without interest-bearing money, though in this case the duration of the monetary transfer is limited by the length of the liquidity trap.

While the effects appear to be somewhat larger than the baseline specification of the model (Figure 4), this mainly reflects the differences in behavior in response to the underlying recessionary shock. In particular, there are three key differences between the two model variants. First, the baseline model exhibits a small increase in money demand as the short-term bond rate is reduced in response to the recessionary shock. Other things equal, higher real money balances support consumption spending via a wealth effect. This means that a somewhat larger shock is required to drive the baseline model to the zero bound. Second, the cash in advance variant implies that steady-state money balances are larger than in the baseline model, so that a given proportionate change in money generates a larger wealth effect on consumption. Third, the cash in advance variant features a cost channel which suppresses inflation when nominal interest rates are low.

To aid the comparison between the two variants, Figure 7 plots the marginal effects of the policy experiments simulated in Figures 4 and 6, normalizing each variable in a way that facilitates the comparison of the effects. We observe from Figure 7 that the stimulatory effects of a money-financed transfer are slightly larger in the cash-in-advance variant of the model, but the pattern and order of magnitude of the effects are very similar.

Figure 7: Marginal effects of money-financed transfers in different model variants



Notes: Each panel plots the marginal effect of the experiments shown in Figures 4 and 6. The dot-dashed black lines depict the responses from Figure 4, using the baseline model. The red dashed lines with diamond markers show the results from 6, using the cash in advance variant. In each case, the marginal effects are computed as the differences between the baseline simulation and the simulation in which a temporary money-financed transfer is undertaken. The right panel shows the marginal effect on the money stock as a fraction of steady-state annual output.

5.2 Additively separable money demand

To illustrate the importance of the monetary frictions, we consider a variant of the model with additively separable money demand. This assumption delivers a tractable specification for money demand, consistent with much of the recent literature (notably Buiter (2005, 2014)). Importantly, it implies that flexible price allocations are independent of monetary developments and hence allows us to specify a monetary policy rule that delivers complete stabilization of the output gap and inflation away from the zero bound. This in turn is important in isolating the importance of the policy rule for the short-term bond rate in determining the effects of monetary transfers. Of course, this variant also has some weaknesses. In particular, the elasticity of money demand to changes in the relative returns on money and bonds is implausibly large.

Appendix D sets out a variant of the model in which (log) real money balances enter the utility function in an additively separable manner. The consumption equation, Phillips curve and monetary policy rule in this case are:

$$
\begin{aligned}
c_t &= (1-\mu_t)^{-1}\frac{\gamma\pi_{t+1}}{R_{t+1}}\left[\frac{\mu_t}{\mu_{t+1}}c_{t+1}+(1-\alpha)\,\mu_t\,(1-\gamma)\,\gamma^{-1}a_{t+1}\right] \\
\frac{\Phi\pi_t}{\pi^* y_t}\left(\frac{\pi_t}{\pi^*}-1\right) &= 1-\eta+\eta\frac{\chi c_t y_t^{\psi}}{1-\alpha}+\Phi\frac{\pi_{t+1}}{R_{t+1}}\frac{\pi_{t+1}}{\pi^* y_t}\left(\frac{\pi_{t+1}}{\pi^*}-1\right) \\
R_{t+1} &= \max\left\{R_{t+1}^{f}\left(\frac{\pi_t}{\pi^*}\right)^{\theta_\pi}\left(\frac{y_t}{y_t^{f}}\right)^{\theta_y},\underline{R}\right\}
\end{aligned}
\tag{19}
$$

where R^f is the nominal interest rate prevailing under flexible prices (derived in Appendix D) and the marginal propensity to consume satisfies:

$$
\mu_t^{-1} = 1+\gamma\beta\frac{\vartheta_{t+1}}{\vartheta_t}\mu_{t+1}^{-1}
$$

The main differences between the baseline model and this variant are that there is no direct impact of money demand (or transactions frictions) in either the Phillips curve or the intertemporal substitution components of the consumption equation.[28] This means that the flexible price allocations can be derived independently of monetary frictions. As a result, it is possible to specify the monetary policy rule so that, away from the lower bound, fluctuations in inflation around target and output around flex-price output are completely stabilized. This is achieved in the rule we use in this model variant via its response to the flexible price interest rate R_{t+1}^{f}.

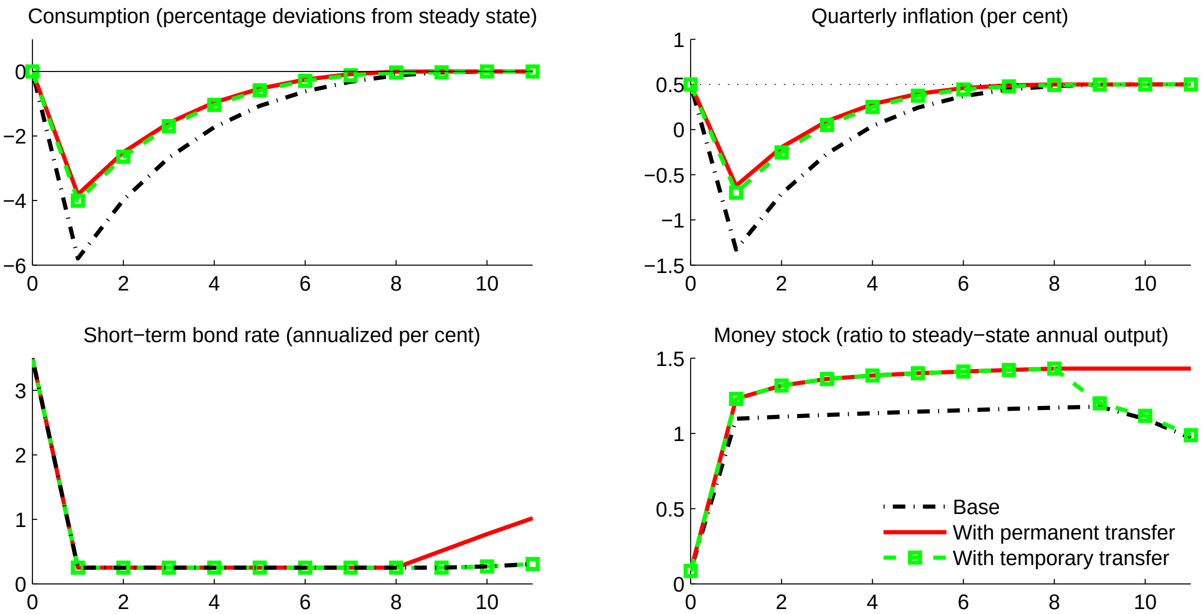
Figure 8 demonstrates the importance of the specification of the Taylor rule by comparing the effects of a temporary and permanent monetary transfer at the lower bound. The baseline (black dash-dot lines) is a recessionary scenario that drives the short-term bond rate to the effective lower bound (which we set at 25bp on an annualized basis). Two money-financed transfers are considered. The solid red lines show the case of a permanent transfer, calibrated in the same way as the experiment in Section 4.3.[29] The dashed green lines with square markers show an 8 quarter temporary transfer of the same size. In each case, for the duration of the transfer, the interest rate on money is adjusted to ensure that the additional money balances are willingly held.

The results show that the additional stimulus from the permanent transfer is extremely small. The reason is that the monetary policy rule (19) delivers complete stabilization of inflation at target when not constrained by the zero bound. The additional stimulus from the permanent money transfer causes the short-term bond rate to lift off from the lower bound earlier. However, inflation is fully stabilized after the liftoff date, limiting the extent to which the real interest rate can be reduced

[28]That is, while assets appear on the right-hand side of the consumption equation, reflecting the net wealth effect, the slope of the consumption equation (in particular the marginal propensity to consume) does not depend on real money balances.

[29]The transfer is calibrated to deliver the same increase in the nominal money stock (measured as a fraction of steady-state GDP) relative to the baseline response of money balances.

Figure 8: Money-financed transfers at the lower bound: additively separable money



Notes: The model starts in period 0 at the steady state and a recessionary shock $\varepsilon_1^{\vartheta} = 0.0185$ arrives in period 1, which drives the economy to the effective lower bound in the baseline simulation (black dash-dot lines) for 9 quarters. Solid red lines show the effect of a permanent monetary transfer and dashed green lines with square markers show the effect of a temporary monetary transfer.

by higher inflation expectations. In contrast, for the baseline model, the results in Figure 5 generated a small but extremely persistent inflation overshoot, depressing real interest rates and providing additional stimulus to spending. That result was driven by the fact that the monetary policy rule in the baseline model does not deliver full stabilization of preference shocks away from the lower bound.

The results in Figure 8 imply that there may be a limit to the degree of stimulus that can be provided by a money-financed transfer, when the money stock and short-term bond rate are both used as policy instruments. For an appropriately specified rule for the short-term bond rate, a money-financed transfer may not generate a *sustained* reduction in real interest rates via a prolonged increase in inflation expectations. In this variant of the model, a permanent money-financed transfer cannot achieve this when the short-term bond rate is determined by (19) because this rule will offset the inflationary effects of the transfer when not constrained by the zero bound.

6 Discussion

In this section we relate our results to the recent debate on the use of monetary-financing to stimulate spending and inflation.

6.1 Interest-bearing money and bank deposits

A key assumption underpinning much of our analysis is that money-financed transfers are implemented by varying the rate of interest paid on money. In this way, government policy can influence the stock of real money balances in the economy while retaining control over the interest rate on short-term bonds because these balances are willingly held by households at the prevailing interest rates on short-term bonds and money.

While our assumption differs from the textbook assumption of non-interest bearing money, we believe it is appropriate in the context of recent discussions of monetary financing. Such discussions often focus on the effects of the increased supply of money generated by such a policy. However, the effects of increased money holdings on spending depend on the extent to which households are willing to hold the additional money balances. For a given rate of return on short-term bonds, ensuring that households willingly hold a particular level of real money balances requires the interest rate on money to adjust.

If money is interpreted as banknotes and coins, then the concept of a variable, non-zero rate of return on money is unrealistic. However, we interpret money in our model in the context of the post-crisis practice of remunerating commercial bank reserves at the policy rate and the fact that the vast majority of the money stock is held in the form of bank deposits (see McLeay et al. (2014)). Many central banks began paying interest on reserves as policy rates approached a (positive) effective lower bound on rates and quantitative easing policies allowed a marked increase in reserves without forcing short-term market rates below that level. Moreover, any monetary transfer from government to households, even if made in the form of notes, would quickly result in an increase in reserves.[30] However, unlike Ireland (2014), we do not explicitly model the demand for bank deposits by households independently of the (derived) demand for reserves by the banking system.

A further implication of paying interest on money is that as that rate approaches the short-term bond rate, money becomes virtually indistinguishable from short-term government debt as a means of financing a deficit. However, our quantitative results hinge on a particular friction that generates net wealth effects – the asset-reset mechanism – which applies equally to bonds and interest-bearing money. Features of real-world economies that are likely to make money-financed transfers effective are also likely to make other policies effective, for example a debt-financed tax cut. This implies that the focus on money financing options as a 'special' policy tool may be misguided, though there are other frictions and attributes of central bank money that could generate a greater efficacy of money financing, as we discuss below.[31]

6.2 Wealth effects via 'irredeemable' money

Several recent papers have explored the notion that central bank money is special because it is viewed as irredeemable by the issuing government (Buiter, 2005, 2014; Buiter and Sibert, 2007). In this approach, households still view interest-bearing money

[30]Households would be expected to deposit their notes in the banking system. Those notes in turn would be exchanged by the banking system for interest-bearing reserves at the central bank.

[31]One reason why monetary financing may be interpreted as 'special' is that, under some circumstances, it operates through mechanisms that could be regarded as unique to monetary policy. For example, monetary financing as modeled by Galí (2014a) implies that government spending is financed using the inflation tax and the resulting high inflation leads to a reduction in real interest rates.

as an asset but, in equilibrium, they do not expect future taxes to be raised to ensure the that the principal of the reserve liability is paid off in present value terms (unlike bonds).[32]

This setup implies an asymmetry in which the present value of the terminal stock of irredeemable reserves will be positive whereas that of redeemable government bonds will be zero. A permanent helicopter drop of money in such an economy could stimulate spending through a type of wealth effect. However for that irredeemability belief to be credible requires a commitment to permanently expand the stock of money at a rate equal to the the nominal interest rate (Buiter, 2014), while expanding the stock of government bonds at a rate below the rate of interest. This setup has several interesting practical implications.

First, a helicopter drop policy of this type would require a shift to a new equilibrium and institutional mode of operation for central banks. Current institutional arrangements imply that central banks and governments do *not* regard money as irredeemable. In our model, the public sector budget constraint is consolidated, so there is no distinction between the balance sheet of the central bank and wider fiscal authority. In practice, however, the payment of interest on reserve liabilities are typically financed by the interest from central bank holdings of government bonds, which in turn are financed by taxes and government borrowing. This implies that money is regarded as redeemable from the perspective of the government.

One way to make money irredeemable would be for the government to cancel the debt used to 'back' reserves. In that case, to implement a helicopter drop, the central bank would need to finance interest payments on reserves by the creation of more reserves. This raises questions of central bank solvency and whether such a concept has any practical relevance (Cumming, 2015; Reis, 2015).

Second, even if such a policy could be implemented, it is not clear that moving to an equilibrium in which the nominal money stock grows at the nominal interest rate is achievable. For example, Buiter and Sibert (2007) present a general equilibrium treatment which shows that the existence of irredeemable money can rule out deflationary bubbles.[33] However, that result also implies that the limiting value of the present discounted value of real money balances is zero *in equilibrium*. This rules out the use of policies that involve expanding the money supply at the rate of interest as advocated by Buiter (2014) in equilibrium.

6.3 Debt versus money finance

One argument in favor of money-financed rather than debt-financed fiscal expansions is that increases in the government debt stock may increase the real interest rates at which the government can finance that debt (see, for example, Smets and Trabandt,

[32]This result holds *in equilibrium* by combining the household sector intertemporal budget constraint (which treats both money and bonds as redeemable) and the government intertemporal budget constraint (in which money is treated as irredeemable).

[33]Transversality conditions on the total value of government liabilities alone are not sufficient to rule out equilibria in which the real values of money and bonds diverge in opposite directions.

2012).[34] Modeling such effects typically involves including a friction whereby some agents have a preference or requirement for holding longer term government liabilities rather short-term liabilities (such as one-period government bonds or money). A long-term bond-financed deficit will increase the term or risk premium on such debt and crowd out private sector spending. Such frictions are similar to those that are often used to motivate a role for quantitative easing (Harrison, 2017, for example). In these case, a money-financed fiscal expansion could be regarded as the equivalent of a bond-financed expansion plus a quantitative easing operation.

These observations raise the possibility that there may be some circumstances in which a combined bond-financed expansion plus a quantitative easing operation would be more effective than either policy alone.[35]

Finally, our quantitative results hinge on the precise friction that generates net wealth effects: the asset-reset mechanism. As explained in Section 2, we use this device for analytical convenience rather than realism. In practice, there are distortions in the economy that are likely to lead to more substantial departures from stark Ricardian equivalence results. For example, in a model without interest on money, Auerbach and Obstfeld (2005) show that monetary expansions at the zero bound can be effective when taxes are distortionary rather than lump sum.

The extent to which different frictions apply to debt versus money financing is likely to have implications for the optimal mix of these financing methods and is a topic that deserves further research.

6.4 Welfare implications

Much of the debate on money-financed fiscal actions takes it for granted that increasing spending and inflation in response to a recessionary shock is welfare improving. However, as Ireland (2005) shows in a similar model, if net wealth effects from real money balances are generated through a redistributional channel then policies that rely heavily on that channel may reduce welfare for many households. Similarly, a money-financed fiscal stimulus implemented in the same way as studied by Galí (2014a) generates a large gap between output and its flexible price counterpart. As Galí (2014a) notes, a more appropriate metric of welfare is the efficient level of output.[36] He shows that, under some conditions, a policy rule in which government spending is financed by money creation can improve welfare. More broadly, the transactions friction in our model would (other things equal) suggest that the returns on money and bonds be equalized, so that transactions costs vanish. A full welfare analysis of money-financed transfers in our model is beyond the scope of the present paper, but

[34] In the limit, it may become impossible for a government to borrow more if its current debt level is sufficiently high.

[35] From the perspective of a simple 'IS-LM' approach, the LM curve may be kinked. A debt-financed fiscal expansion alone pushes up on the real interest rate and crowds out the stimulative effect. A simultaneous quantitative easing operation moves the kink in the LM curve, so that a debt-financed fiscal expansion is not associated with extreme crowding out.

[36] That is, the level of output that would prevail when distortions from monopolistic competition are eliminated.

is an interesting topic for future research.

6.5 Permanent liquidity trap versus temporary lower bound episode

Our focus in this paper is on the potential efficacy of money-financed fiscal policy during a period in which the short-term interest rate is temporarily constrained by the effective lower bound. This is in line with most of the literature studying policy options at the zero lower bound, much of it inspired by the seminal contributions of Krugman (1998) and Eggertsson and Woodford (2003). It is also consistent with much of the recent commentary on the use of helicopter drops as a temporary measure to provide stimulus.[37]

The extent to which monetary-financing may be useful in a permanent liquidity trap, in which the short-term bond rate remains at the zero bound forever is analyzed by Buiter (2014) in a partial equilibrium context. Ireland (2005) studies the property of a very similar model in a liquidity trap environment, but in his case the liquidity trap is a policy choice.[38] Again, we regard an assessment of the efficacy of money-financed fiscal policy in a permanent liquidity trap as an interesting avenue for future research.

7 Conclusion

We assess recent proposals for the use of money-financed transfers to stimulate economic activity and inflation using a simple sticky-price model. We examine the efficacy of these policies in the context of a recessionary shock that temporarily forces the short-term policy rate to the effective lower bound. Our model allows for net wealth effects and, in this case, money-financed transfers can stimulate spending and inflation. However, the scale of the transfers required to generate meaningful effects is very large and could also be achieved by a bond-financed deficit.

In our model, money may earn a non-zero rate of return and may therefore be interpreted as a digital currency rather than cash. Exploring this interpretation of our framework is an interesting topic for future research.

[37] Though Turner (2015) argues that monetary financing may become a conventional monetary policy tool in a world of secular stagnation.

[38] The short-term nominal interest rate remains permanently at zero if the government chooses to expand the money stock at a sufficiently low rate.

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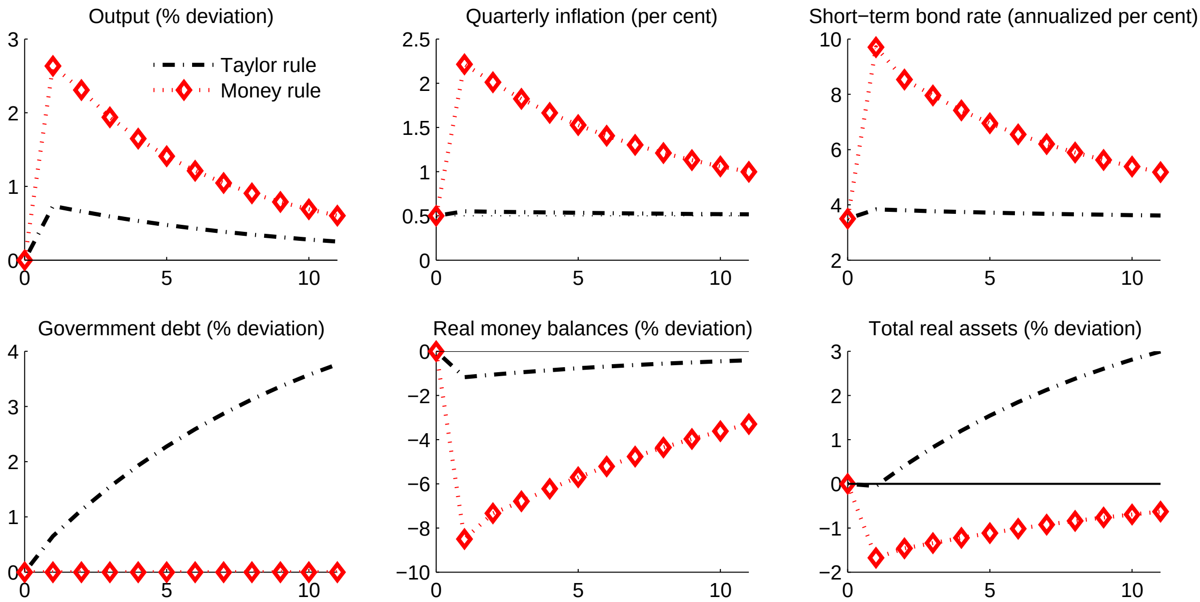
A Additional results

A.1 Money vs debt financing when $\gamma = 1$

Figure A.1 shows the results of replicating the experiment in Figure 1 when we abstract entirely from real balance effects by setting $\gamma = 1$. The results are virtually identical to those shown in Figure 1.

Figure A.1: Money-financed and debt-financed government spending with $\gamma = 1$



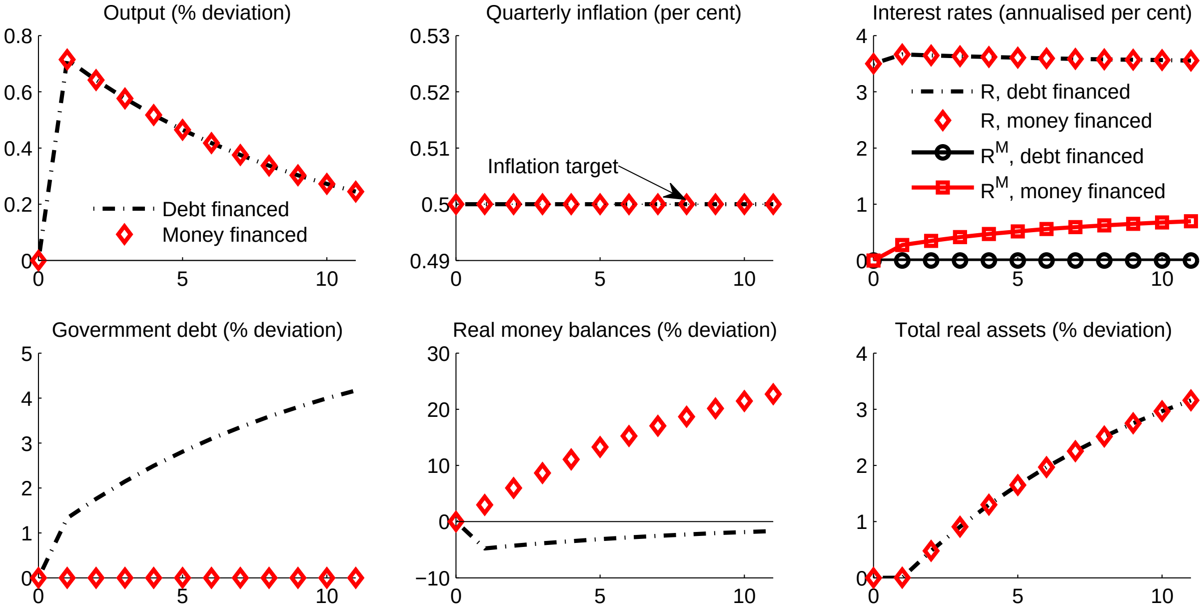
Notes: The model starts in period 0 at the steady state. Government spending is given by (9). The panels show the responses to $\varepsilon_1^g = 0.05$ (with $\varepsilon_t^g = 0, t \geq 2$). In the Taylor rule case, the tax rate fiscal rule is initially inactive so that $\tau_t = \tau^*$ for $t = 1, \ldots, K$, where $K = 12$. From period $t = K + 1$ taxes are set according to (15) with $\theta_b = 0.1$. The short-term interest rate is set according to (7). In the money rule case, the tax rate and debt stock are held constant and the government spending increase is financed by money creation according to equation (16). There is no interest paid on money ($R_t^M = 1, \forall t$) so the short term interest rate R_t adjusts to ensure that (16) holds. The experiment is conducted in a version of the model without asset resets (i.e., $\gamma = 1$).

A.2 Additively separable money demand

In this Appendix we report the results of the experiment shown in Figure 2 in a version of the model with additively separable money demand (derived in Appendix D) and without asset resets ($\gamma = 1$). This variant is closest to the one analyzed by Galí (2014a), with the key difference being that money is interest bearing.

Figure A.2 repeats the experiment shown in Figure 2. As in Figure 2, the government spending increase is financed either using short-term debt (with the short-term nominal interest rate adjusting according to the Taylor rule) or by interest-bearing money (with the interest rate on money adjusting to ensure that households willingly hold the additional real money balances). We observe that, in the variant with additively separable money demand, the outcomes for output, inflation and total real assets are identical.

Figure A.2: Financing government spending with interest-bearing money: additively separable case



Notes: The model starts in period 0 at the steady state. Government spending is given by (9). The panels show the responses to $\varepsilon_1^g = 0.05$ (with $\varepsilon_t^g = 0, t = 2, \ldots$). In the debt financed case, the tax rate fiscal rule is initially inactive so that $\tau_t = \tau^*$ for $t = 1, \ldots, K$, where $K = 12$. From period $t = K + 1$ taxes are set according to (15) with $\theta_b = 0.1$. In the money financed case, the tax rate and debt stock are held constant and the government spending increase is financed by money creation according to equation (16). The return on money is adjusted to ensure that households willingly hold the additional money balances. The experiment is conducted in a version of the model with (additively separable) money in the utility function and without asset resets.

B Derivation of the baseline model

B.1 Households

The maximization problem is:

$$\max \sum_{t=0}^{\infty} (\gamma\beta)^t \vartheta_t \left[\ln c_{j,t} - \frac{\chi_{j,t}}{1+\psi} n_{j,t}^{1+\psi}\right]$$

subject to

$$m_{j,t}^p + b_{j,t}^p = (\gamma\pi_t)^{-1}\left[R_t^M m_{j,t-1}^p + R_t b_{j,t-1}^p\right] + w_t n_{j,t} + d_t - \tau_t - \left(1 + \varphi\left(\frac{c_{j,t}}{m_{j,t}^p}\right)\right) c_{j,t}$$

where we write the budget constraint in real terms and

$$\varphi\left(v_{j,t}\right) = Z \exp\left[-\frac{\zeta}{v_{j,t}}\right]$$

$Z, \zeta > 0$.

The first order conditions are:

$$\begin{aligned}
\lambda_{j,t} - \beta\pi_{t+1}^{-1}R_{t+1}\lambda_{j,t+1} &= 0\\
\vartheta_t c_{j,t}^{-1} - \lambda_{j,t}\left(1+\varphi\left(v_{j,t}\right)+\varphi'\left(v_{j,t}\right)v_{j,t}\right) &= 0\\
\lambda_{j,t}\left(1-\varphi'\left(v_{j,t}\right)v_{j,t}^2\right) - \beta\pi_{t+1}^{-1}R_{t+1}^M\lambda_{j,t+1} &= 0\\
\chi_{j,t}\vartheta_t n_{j,t}^{\psi} - \lambda_{j,t}w_t &= 0
\end{aligned}$$

This functional form of φ implies that

$$\varphi'\left(v_{j,t}\right) = \varphi\left(v_{j,t}\right)\zeta v_{j,t}^{-2}$$

This implies that the set of first order conditions can be written as:

$$\lambda_{j,t} - \beta\pi_{t+1}^{-1}R_{t+1}\lambda_{j,t+1} = 0 \tag{B.1}$$
$$\vartheta_{j,t} c_{j,t}^{-1} - \lambda_{j,t}\left[1+\varphi\left(v_{j,t}\right)\left(1+\zeta v_{j,t}^{-1}\right)\right] = 0 \tag{B.2}$$
$$\lambda_{j,t}\left(1-\zeta\varphi\left(v_{j,t}\right)\right) - \beta\pi_{t+1}^{-1}R_{t+1}^M\lambda_{j,t+1} = 0 \tag{B.3}$$
$$\chi_{j,t}\vartheta_t n_{j,t}^{\psi} - \lambda_{j,t}w_t = 0 \tag{B.4}$$

Combining the first order conditions for bonds and money gives:

$$1-\zeta\varphi\left(v_{j,t}\right) = \frac{R_{t+1}^M}{R_{t+1}}$$

which shows that velocity is determined entirely by the difference between the rates of return on money and bonds. Since these rates of return are the same for all households, velocity is the same for each household:

$$v_{j,t} = v_t \quad \forall j,t$$

where v_t denotes aggregate velocity (total consumption divided by total real money balances). We impose this result in the rest of the derivation.

The previous result means that

$$\varphi\left(v_t\right) = \zeta^{-1}\frac{R_{t+1}-R_{t+1}^M}{R_{t+1}}$$

which in turn implies that

$$\ln Z - \frac{\zeta}{v_t} = -\ln\zeta + \ln\frac{R_{t+1}-R_{t+1}^M}{R_{t+1}}$$

so that the demand for real money balances is given by:

$$m^p_{j,t} = \zeta^{-1}\left[\ln(\zeta Z) - \ln\frac{R_{t+1} - R^M_{t+1}}{R_{t+1}}\right] c_{j,t}$$

which implies that aggregate money demand satisfies

$$m_t = \zeta^{-1}\left[\ln(\zeta Z) - \ln\frac{R_{t+1} - R^M_{t+1}}{R_{t+1}}\right] c_t$$

Rearranging the first order condition for consumption gives:

$$\lambda_{j,t} = \frac{\vartheta_t}{c_{j,t}\left[1 + \varphi(v_t)\left(1 + \zeta v_t^{-1}\right)\right]}$$

which we can combine with the first order condition for bonds to give an Euler equation

$$\widetilde{c}_{j,t+1} = \beta\pi^{-1}_{t+1}R_{t+1}\frac{\widetilde{\vartheta}_{t+1}}{\widetilde{\vartheta}_t}\widetilde{c}_{j,t} \tag{B.5}$$

where $\widetilde{c}_{j,t}$ denotes consumption inclusive of transactions costs,

$$\widetilde{c}_t \equiv (1 + \varphi(v_t))\, c_{j,t}$$

and

$$\widetilde{\vartheta}_t \equiv \frac{\vartheta_t (1 + \varphi(v_t))}{1 + \varphi(v_t)\left(1 + \zeta v_t^{-1}\right)}$$

Combining the first order conditions for consumption and hours worked gives:

$$\chi_{j,t} n^{\psi}_{j,t} = \frac{w_t}{c_{j,t}\left[1 + \varphi(v_t)\left(1 + \zeta v_t^{-1}\right)\right]}$$

Given the specification of $\chi_{j,t}$ we have:

$$\chi n^{\psi}_{j,t} = \frac{w_t}{c_t\left[1 + \varphi(v_t)\left(1 + \zeta v_t^{-1}\right)\right]}$$

B.2 Derivation of the aggregate consumption equation

For the derivation it is useful to define a household's total financial assets:

$$A^p_{j,t+1} \equiv R^M_{t+1} M^p_{j,t} + R_{t+1} B^p_{j,t} \tag{B.6}$$

which represents the total effective monetary and non-monetary obligations of the government, including interest due, in period $t+1$.

Using the definition of assets and $\widetilde{c}$, the household budget constraint can be written as:

$$\begin{aligned}\frac{A^{p}_{j,t+1}}{P_t R_{t+1}} + \frac{M^{p}_{j,t}}{P_t} - \frac{R^{M}_{t+1} M^{p}_{j,t}}{R_{t+1} P_t} &= \frac{A^{p}_{j,t}}{\gamma P_t} + \tilde{w}_{j,t} - \widetilde{c}_{j,t} \\ \frac{a^{p}_{j,t+1}\pi_{t+1}}{R_{t+1}} + \frac{\left(R_{t+1} - R^{M}_{t+1}\right) M^{p}_{j,t}}{R_{t+1} P_t} &= \gamma^{-1} a^{p}_{j,t} + \tilde{w}_{j,t} - \widetilde{c}_{j,t}\end{aligned}$$

where the first line uses the fact that (B.6) implies that $B^{p}_{j,t} = R^{-1}_{t+1} A^{p}_{j,t+1} - R^{-1}_{t+1} R^{M}_{t+1} M^{p}_{j,t+1}$ and the second line uses the definition of real assets $a^{p}_{j,t} \equiv A^{p}_{j,t}/P_t$ and the inflation rate $\pi_t \equiv P_t/P_{t-1}$. The final line can be written as:

$$a^{p}_{j,t} = \frac{\gamma a^{p}_{j,t+1}\pi_{t+1}}{R_{t+1}} + \gamma \left[\widetilde{c}_{j,t} - \tilde{w}_{j,t} + \frac{R_{t+1} - R^{M}_{t+1}}{R_{t+1}} m^{p}_{j,t}\right] \tag{B.7}$$

The real discount factor is defined recursively as:

$$\mathcal{D}_{t+i} = \frac{\gamma \pi_{t+i}}{R_{t+i}} \mathcal{D}_{t+i-1} \tag{B.8}$$

from $\mathcal{D}_t = 1$.

The household's no-Ponzi condition is assumed to be:

$$\lim_{i\to\infty} \mathcal{D}_{t+i} a^{p}_{j,t+i} \geq 0 \tag{B.9}$$

Iterating the household budget constraint (B.7) gives:

$$a^{p}_{j,t} = \lim_{i\to\infty} \mathcal{D}_{t+i} \frac{\gamma a^{p}_{j,t+i}\pi_{t+i+1}}{R_{t+i+1}} + \gamma \sum_{i=0}^{\infty} \mathcal{D}_{t+i} \left(\widetilde{c}_{j,t+i} - \tilde{w}_{j,t+i} + \frac{R_{t+i+1} - R^{M}_{t+i+1}}{R_{t+i+1}} m^{p}_{j,t+i}\right)$$

so that, if the no Ponzi constraint binds with equality (as it will if marginal utility is positive in the limit):

$$a^{p}_{j,t} = \gamma \sum_{i=0}^{\infty} \mathcal{D}_{t+i} \left(\widetilde{c}_{j,t+i} - \tilde{w}_{j,t+i} + \frac{R_{t+i+1} - R^{M}_{t+i+1}}{R_{t+i+1}} m^{p}_{j,t+i}\right) \tag{B.10}$$

Using the definition of post-tax income in (B.10) gives:

$$a^{p}_{j,t} = \gamma \sum_{i=0}^{\infty} \mathcal{D}_{t+i} \left(\widetilde{c}_{j,t+i} - w_{t+i} n_{j,t+i} - d_{j,t+i} + \tau_{j,t+i} + \frac{R_{t+i+1} - R^{M}_{t+i+1}}{R_{t+i+1}} m^{p}_{j,t+i}\right) \tag{B.11}$$

We can substitute the household's money demand equation into (B.11) to give

$$a^{p}_{j,t} = \gamma \sum_{i=0}^{\infty} \mathcal{D}_{t+i} \left(\widetilde{c}_{j,t+i} - w_{t+i} n_{j,t+i} - d_{j,t+i} + \tau_{j,t+i} + \Xi_{t+i}\widetilde{c}_{j,t+i}\right)$$

where

$$\Xi_{t+i} \equiv \frac{R_{t+i+1} - R^M_{t+i+1}}{R_{t+i+1}} \left(1 + \varphi\left(v_{t+i}\right)\right)^{-1} \zeta^{-1} \left[\ln\left(\zeta Z\right) - \ln \frac{R_{t+i+1} - R^M_{t+i+1}}{R_{t+i+1}}\right]$$

is determined by the relative rates of return on bonds and money.

Rearranging the intertemporal budget constraint gives:

$$\sum_{i=0}^{\infty} \mathcal{D}_{t+i} \left[1 + \Xi_{t+i}\right] \widetilde{c}_{j,t+i} = \gamma^{-1} a^p_{j,t} + \sum_{i=0}^{\infty} \mathcal{D}_{t+i} \left(w_{t+i} n_{j,t+i} + d_{j,t+i} - \tau_{j,t+i}\right) \tag{B.12}$$

The Euler equation (B.5) implies that

$$\widetilde{c}_{j,t+i} = \left(\gamma\beta\right)^i \mathcal{D}^{-1}_{t+i} \frac{\widetilde{\vartheta}_{t+i}}{\widetilde{\vartheta}_t} \widetilde{c}_{j,t} \tag{B.13}$$

Using (D.8) allows us to write (D.7) in terms of current consumption:

$$\widetilde{c}_{j,t} = \mu_t \left[\gamma^{-1} a^p_{j,t} + \sum_{i=0}^{\infty} \mathcal{D}_{t+i} \left(w_{t+i} n_{j,t+i} + d_{j,t+i} - \tau_{j,t+i}\right)\right] \tag{B.14}$$

where μ is the marginal propensity to consume from wealth, given by:

$$\mu_t = \left(\sum_{i=0}^{\infty} \left(\gamma\beta\right)^i \frac{\widetilde{\vartheta}_{t+i}}{\widetilde{\vartheta}_t} \left[1 + \Xi_{t+i}\right]\right)^{-1} \tag{B.15}$$

which implies that:

$$\mu^{-1}_t = 1 + \Xi_t + \gamma\beta \frac{\widetilde{\vartheta}_{t+1}}{\widetilde{\vartheta}_t} \mu^{-1}_{t+1} \tag{B.16}$$

The consumption function (D.9) now aggregates straightforwardly. This follows from the fact that future income flows are identical for all households. Identical income flows are delivered by the assumption of identical lump sum taxes and dividends for all households together with our specification of the preference shifter $\chi_{j,t}$ to eliminate cohort-specific labor supply effects.

This implies that

$$\widetilde{c}_t = \mu_t \left[a_t + \sum_{i=0}^{\infty} \mathcal{D}_{t+i} \left(w_{t+i} n_{t+i} + d_{t+i} - \tau_{t+i}\right)\right] \tag{B.17}$$

where we also impose market clearing for assets. The coefficient on assets on the right hand side of (B.17) is unity (rather that γ^{-1}) because it represents a weighted average of the assets held by households that experience an asset reset and those that do not. The former group has a weight of $1 - \gamma$ and hold no assets. The latter group has a weight γ and average asset holdings of $\gamma^{-1} a_t$.

Substituting the definition of dividends into the consumption function gives:

$$\begin{aligned}\widetilde{c}_t =& \mu_t \left[a_t + \sum_{i=0}^{\infty} \mathcal{D}_{t+i} \left(y_{t+i} - \frac{\Phi}{2}\left(\frac{\pi_{t+i}}{\pi^*} - 1\right)^2 - \tau_{t+i}\right)\right] \\ =& \mu_t \left[a_t + \sum_{i=0}^{\infty} \mathcal{D}_{t+i} \left(\widetilde{c}_{t+i} + g_{t+i} - \tau_{t+i}\right)\right]\end{aligned}$$

where the second line uses the market clearing condition to substitute for output.

We can use the final equation to note that aggregate consumption in period $t+1$ is given by:

$$\widetilde{c}_{t+1} = \mu_{t+1} \left[a_{t+1} + \mathcal{D}_{t+1}^{-1} \sum_{i=1}^{\infty} \mathcal{D}_{t+i} \left(\widetilde{c}_{t+i} + g_{t+i} - \tau_{t+i}\right)\right]$$

The consumption functions at dates $t+1$ and t can be combined to eliminate discounted future income flows:

$$\begin{aligned}\mathcal{D}_{t+1} \frac{\mu_t}{\mu_{t+1}} \widetilde{c}_{t+1} - \widetilde{c}_t =& \mathcal{D}_{t+1} \mu_t a_{t+1} - \mu_t a_t - \mu_t \left(\widetilde{c}_t + g_t - \tau_t\right) \\ =& \mathcal{D}_{t+1} \mu_t a_{t+1} - \mu_t \left(\widetilde{c}_t + a_t + g_t - \tau_t\right)\end{aligned} \tag{B.18}$$

To proceed, we combine the intertemporal budget constraints of the household and the government. In parallel with households, we define the total stock of government liabilities as

$$A_{t+1}^g \equiv R_{t+1}^M M_t^g + R_{t+1} B_t^g$$

Again using lower case notation to denote real-valued asset stocks, we can use these definitions to write the government budget constraint (8) as:

$$a_t^g = \frac{a_{t+1}^g \pi_{t+1}}{R_{t+1}} - g_t + \tau_t + \frac{R_{t+1} - R_{t+1}^M}{R_{t+1}} m_t^g \tag{B.19}$$

This implies that

$$a_t + g_t - \tau_t = \gamma^{-1} \mathcal{D}_{t+1} a_{t+1} + \Xi_t \widetilde{c}_t$$

which we can use in (B.18) to give:

$$\mathcal{D}_{t+1} \frac{\mu_t}{\mu_{t+1}} \widetilde{c}_{t+1} - \widetilde{c}_t = \mathcal{D}_{t+1} \mu_t a_{t+1} - \mu_t \left(\widetilde{c}_t + \gamma^{-1} \mathcal{D}_{t+1} a_{t+1} + \Xi_t \widetilde{c}_t\right)$$

The government budget constraint (B.19), can be written in aggregate terms (imposing asset market equilibrium $a_t^g = a_t^p = a_t, \forall t$) as:

$$\begin{aligned} a_t =& \mathcal{D}^g_{t+1} a_{t+1} - g_t + \tau_t + \frac{R_{t+1} - R^M_{t+1}}{R_{t+1}} m_t \\ =& \gamma^{-1} \mathcal{D}_{t+1} a_{t+1} - g_t + \tau_t + \frac{R_{t+1} - R^M_{t+1}}{R_{t+1}} m_t \\ =& \gamma^{-1} \mathcal{D}_{t+1} a_{t+1} - g_t + \tau_t + \Xi_t \widetilde{c}_t \end{aligned}$$

where the final line substitutes for money demand.

Collecting terms gives:

$$\mathcal{D}_{t+1} \frac{\mu_t}{\mu_{t+1}} \widetilde{c}_{t+1} = \mathcal{D}_{t+1} \mu_t \left(1 - \gamma^{-1}\right) a_{t+1} + \left[1 - \left(1 + \Xi_t\right) \mu_t\right] \widetilde{c}_t$$

which implies that

$$\widetilde{c}_t = \left[1 - \left(1 + \Xi_t\right) \mu_t\right]^{-1} \frac{\gamma \pi_{t+1}}{R_{t+1}} \left[\frac{\mu_t}{\mu_{t+1}} \widetilde{c}_{t+1} + \mu_t \left(1 - \gamma\right) \gamma^{-1} a_{t+1} \right] \tag{B.20}$$

where we also use the fact that $\mathcal{D}_{t+1} = \frac{\gamma \pi_{t+1}}{R_{t+1}}$.

When $\gamma = 1$, equation (B.20) becomes:

$$\widetilde{c}_t = \left[1 - \left(1 + \Xi_t\right) \mu_t\right]^{-1} \frac{\pi_{t+1}}{R_{t+1}} \frac{\mu_t}{\mu_{t+1}} \widetilde{c}_{t+1}$$

and the dependence on assets a_{t+1} (the real balance effect) disappears.

The terms in the marginal propensities to consume can be simplified as follows:

$$\begin{aligned} \left[1 - \left(1 + \Xi_t\right) \mu_t\right]^{-1} \frac{\mu_t}{\mu_{t+1}} &= \left[\frac{\mu_{t+1}}{\mu_t} \left(1 - \left(1 + \Xi_t\right) \mu_t\right) \right]^{-1} \\ &= \left[\frac{\mu_t^{-1}}{\mu_{t+1}^{-1}} - \frac{1 + \Xi_t}{\mu_{t+1}^{-1}} \right]^{-1} \\ &= \left[\frac{1 + \Xi_t + \gamma \beta \frac{\widetilde{\vartheta}_{t+1}}{\widetilde{\vartheta}_t} \mu_{t+1}^{-1}}{\mu_{t+1}^{-1}} - \frac{1 + \Xi_t}{\mu_{t+1}^{-1}} \right]^{-1} \\ &= \left[\gamma \beta \frac{\widetilde{\vartheta}_{t+1}}{\widetilde{\vartheta}_t} \right]^{-1} \end{aligned}$$

Similarly,

$$\left[1 - \left(1 + \Xi_t\right) \mu_t\right]^{-1} \mu_t = \left[\gamma \beta \frac{\widetilde{\vartheta}_{t+1}}{\widetilde{\vartheta}_t} \right]^{-1} \mu_{t+1}$$

Using these results in the aggregate consumption equation gives

$$\widetilde{c}_t = \frac{\pi_{t+1}}{\beta R_{t+1}} \frac{\widetilde{\vartheta}_t}{\widetilde{\vartheta}_{t+1}} \left[\widetilde{c}_{t+1} + (1-\gamma)\,\gamma^{-1}\mu_{t+1}a_{t+1}\right]$$

or

$$\widetilde{c}_t = \frac{\pi_{t+1}}{\beta R_{t+1}} \frac{\widetilde{\vartheta}_t}{\widetilde{\vartheta}_{t+1}} \left[\widetilde{c}_{t+1} + (1-\gamma)\,\gamma^{-1}\mu_{t+1}\pi_{t+1}^{-1}\left(R_{t+1}^{M}m_t + R_{t+1}b_t\right)\right] \tag{B.21}$$

where we have used the definition of total assets:

$$a_{t+1} = \pi_{t+1}^{-1}\left(R_{t+1}^{M}m_t + R_{t+1}b_t\right)$$

When $\gamma = 1$, this can be written as:

$$\widetilde{c}_{t+1} = \beta\pi_{t+1}^{-1}R_{t+1}\frac{\widetilde{\vartheta}_{t+1}}{\widetilde{\vartheta}_t}\widetilde{c}_t$$

which coincides with the individual household Euler equation (B.5).

B.3 Firms and supply side

The objective function for the firm is:

$$\max \sum_{k=t}^{\infty} \bar{\lambda}_k \beta^{k-t}\left[\left(\frac{P_{j,k}}{P_k} - \frac{w_k}{A}\right)\left(\frac{P_{j,k}}{P_k}\right)^{-\eta} y_k - \frac{\Phi}{2}\left(\frac{P_{j,k}}{\pi^* P_{j,k-1}} - 1\right)^2\right]$$

where $\bar{\lambda}$ is a stochastic discount factor used to value the flow of profits. In general, there are heterogeneous households (which differ by their date of asset resets) so there is not a unique stochastic discount factor that can be used for valuing the profit flows paid to households. However, the first order condition (B.1) implies that the discount factors of all households satisfy:

$$\frac{\lambda_{j,t}}{\lambda_{j,t+1}} = \beta\frac{R_{t+1}}{\pi_{t+1}}$$

so we choose a discount factor that satisfies:

$$\bar{\lambda}_{t+1} = \bar{\lambda}_t \frac{\pi_{t+1}}{\beta R_{t+1}}$$

The first order condition for the firm's price is:

$$\begin{aligned} 0 = &-\eta\left(\frac{P_{j,t}}{P_t} - \frac{w_t}{A}\right)\left(\frac{P_{j,t}}{P_t}\right)^{-\eta-1}\frac{\bar{\lambda}_t y_t}{P_t} + \left(\frac{P_{j,t}}{P_t}\right)^{-\eta}\frac{\bar{\lambda}_t y_t}{P_t} \\ &- \frac{\Phi\bar{\lambda}_t}{\pi^* P_{j,t-1}}\left(\frac{P_{j,t}}{\pi^* P_{j,t-1}} - 1\right) + \beta\frac{\Phi\bar{\lambda}_{t+1}P_{j,t+1}}{\pi^* P_{j,t}^2}\left(\frac{P_{j,t+1}}{\pi^* P_{j,t}} - 1\right) \end{aligned}$$

which reveals that optimal pricing decisions depend on the stochastic discount factor only through the ratio $\bar{\lambda}_{t+1}/\bar{\lambda}_t$.

In a symmetric equilibrium in which $P_{j,t} = P_t, \forall j, t$, the first order condition simplifies to:

$$\frac{\Phi\pi_t}{\pi^* y_t}\left(\frac{\pi_t}{\pi^*}-1\right) = 1-\eta+\eta\frac{w_t}{A}+\Phi\frac{\pi_{t+1}}{R_{t+1}}\frac{\pi_{t+1}}{\pi^* y_t}\left(\frac{\pi_{t+1}}{\pi^*}-1\right) \tag{B.22}$$

The supply side of the model is unchanged from the baseline variant, so the pricing equation is given by:

$$\frac{\Phi\pi_t}{\pi^* y_t}\left(\frac{\pi_t}{\pi^*}-1\right) = 1-\eta+\eta\frac{w_t}{A}+\Phi\frac{\pi_{t+1}}{R_{t+1}}\frac{\pi_{t+1}}{\pi^* y_t}\left(\frac{\pi_{t+1}}{\pi^*}-1\right)$$

as before.

Noting that $A = 1$ and that the first order conditions imply that the wage is given by:

$$w_t = \chi n_t^{\psi} c_t\left[1+\varphi\left(v_t\right)\left(1+\zeta v_t^{-1}\right)\right]$$

allows us to write the pricing equation as:

$$\frac{\Phi\pi_t}{\pi^* y_t}\left(\frac{\pi_t}{\pi^*}-1\right) = 1-\eta+\eta\chi y_t^{\psi} c_t\left[1+\varphi\left(v_t\right)\left(1+\zeta v_t^{-1}\right)\right]+\Phi\frac{\pi_{t+1}}{R_{t+1}}\frac{\pi_{t+1}}{\pi^* y_t}\left(\frac{\pi_{t+1}}{\pi^*}-1\right)$$

where we also use the fact that the production function implies that $n_t = y_t$.

B.4 The model equations

Collecting together the previously derived equations gives:

$$\begin{aligned}
y_t &= (1+\varphi_t)\, c_t + g^* + \frac{\Phi}{2}\left(\frac{\pi_t}{\pi^*}-1\right)^2 \\
\frac{\Phi\pi_t}{y_t\pi^*}\left(\frac{\pi_t}{\pi^*}-1\right) &= 1-\eta+\eta\chi y_t^{\psi} c_t\left[1+\varphi_t\left(1+\zeta v_t^{-1}\right)\right]+\Phi\frac{\pi_{t+1}}{R_{t+1}}\frac{\pi_{t+1}}{\pi^* y_t}\left(\frac{\pi_{t+1}}{\pi^*}-1\right) \\
\widetilde{c}_t &= \frac{\pi_{t+1}}{\beta R_{t+1}}\frac{\widetilde{\vartheta}_t}{\widetilde{\vartheta}_{t+1}}\left[\widetilde{c}_{t+1}+(1-\gamma)\,\gamma^{-1}\mu_{t+1}\pi_{t+1}^{-1}\left(R_{t+1}^{M} m_t + R_{t+1} b_t\right)\right] \\
\mu_t^{-1} &= 1+\Xi_t+\gamma\beta\frac{\widetilde{\vartheta}_{t+1}}{\widetilde{\vartheta}_t}\mu_{t+1}^{-1} \\
\Xi_t &= \frac{\varphi_t}{1+\varphi_t}\left[\ln(\zeta Z)-\ln\frac{R_{t+1}-R_{t+1}^{M}}{R_{t+1}}\right] \\
\widetilde{c}_t &= (1+\varphi_t)\, c_t \\
\widetilde{\vartheta}_t &= \frac{\vartheta_t\,(1+\varphi_t)}{1+\varphi_t\left(1+\zeta v_t^{-1}\right)} \\
\varphi_t &= \zeta^{-1}\frac{R_{t+1}-R_{t+1}^{M}}{R_{t+1}} \\
\varphi_t &= Z\exp\left[-\frac{\zeta}{v_t}\right] \\
v_t &= \frac{c_t}{m_t} \\
R_{t+1} &= \max\left\{R\left(\frac{\pi_t}{\pi^*}\right)^{\theta_\pi}\left(\frac{y_t}{y_t^{f}}\right)^{\theta_y}, \underline{R}\right\} \\
\Delta\ln\vartheta_t &= \rho_\vartheta\Delta\ln\vartheta_{t-1}+\varepsilon_t^{\vartheta}
\end{aligned}$$

Conditional on the behavior of flex price output, y_t^f (derived below), the return on money, R_t^M, and government debt, b_t, this system determines the variables $y_t, c_t, \pi_t, R_{t+1}, m_t, v_t, \varphi_t, \vartheta_t, \widetilde{\vartheta}_t, \widetilde{c}_t, \mu_t, \Xi_t$.

The model is closed by assumptions about the return on money and government debt. The gross rate of return on money, R_{t+1}^M, is either held fixed at unity in the conventional approach ($R_{t+1}^M = 1, \forall t$) or adjusted to ensure that households willingly hold a stock of money that is determined by a rule, as in the baseline version of the model. The baseline assumption for government debt is that it is fixed, $b_t = b^* \geq 0$, while the 'debt financing' experiments in Section 3.1 incorporate the government budget constraint with temporarily fixed taxes, $\tau_t = \tau$.

B.5 Flexible price allocations

Models with transactions frictions present some challenges for defining flexible price allocations. In particular, even if prices and wages are fully flexible, allocations still depend on the level(s) of nominal interest rate(s). We follow Kim and Subramanian (2006) and Ravenna and Walsh (2006) and define a 'supply side' flexible price equilibrium, conditional on steady-state nominal returns on money and bonds.

If nominal returns on bonds and money are at their steady-state levels, of R and 1 respectively, then flexible price velocity and transactions costs are constant, at $\bar{\varphi}$ and $\bar{v}$ respectively, where these values satisfy:

$$\zeta\bar{\varphi} = \frac{R-1}{R}$$
$$\bar{\varphi} = Z\exp\left[-\frac{\zeta}{\bar{v}}\right]$$

Conditional on flexible price velocity and transactions costs, market clearing and the pricing equation can be used to solve for flexible price output and consumption as follows:

$$y_t^f = (1+\bar{\varphi})\,c_t^f + g_t$$
$$0 = 1-\eta+\eta\chi c_t^f\left(y_t^f\right)^{\psi}\left(1+\bar{\varphi}\left(1+\frac{\zeta}{\bar{v}}\right)\right)$$

where the pricing equation does not feature price adjustment costs under the assumption that inflation is constant at target in the flexible price equilibrium.

B.6 Steady state

We use a standard notation in which steady-state allocations are indicated by the absence of time subscripts.

Long-run government policies are treated as exogenous. This includes the value of the inflation target π^* and the steady-state levels of debt and government spending. We assume that steady-state government spending is set exogenously at $g = g^* y$, with $g^* \in [0,1)$.

We choose the utility parameter χ and the productivity parameter A to normalize steady-state hours worked and output at unity: $n = y = 1$. Conditional on $n = 1$, the required value of A is 1. The required value of χ is found by noting that the steady-state Phillips curve implies that:

$$0 = 1-\eta+\eta\chi\frac{1-g^*}{1+\varphi}\left[1+\varphi\left(1+\zeta v^{-1}\right)\right]$$

which means that we require:

$$\chi = \frac{\eta-1}{\eta}\frac{1+\varphi}{(1-g^*)\left[1+\varphi\left(1+\zeta v^{-1}\right)\right]}$$

which means that the required value of χ depends on steady state velocity and (hence) transactions costs. These variables will be targeted by an appropriate choice of the transaction cost parameters Z and ζ (detailed below).

In steady state, the consumption equation implies

$$\beta\widetilde{c} = \frac{\pi^*}{R}\left[\widetilde{c} + \mu\left(1-\gamma\right)\frac{1}{\gamma\pi^*}\left(R^M m + Rb\right)\right]$$

or

$$\frac{\beta R}{\pi^*} = 1 + \frac{1-\gamma\beta}{1+\Xi}\left(1-\gamma\right)\frac{R^M m + Rb}{\gamma\pi^*\widetilde{c}}$$

(since $\mu = \frac{1-\gamma\beta}{1+\Xi}$). This illustrates that the steady state real interest rate is increasing in the steady state ratio of (the value of) government liabilities to consumption.

The steady-state net interest rate on money is assumed to be zero so that $R^M = 1$. As noted above, we also treat steady-state velocity, $v \equiv c/m$ as a calibration target and is therefore 'known'.

This means that the steady-state interest rate satisfies:

$$\frac{\beta R}{\pi^*} = 1 + \frac{\left(1-\gamma\beta\right)\left(1-\gamma\right)}{\left(1+\Xi\right)\left(1+\varphi\right)v\gamma\pi^*} + \frac{1-\gamma\beta}{1+\Xi}\left(1-\gamma\right)\frac{Rb}{\gamma\pi^*\widetilde{c}}$$

We assume that the steady-state target level of government debt is given by $b = b^* y$, where $b^* \geq 0$ and $y = 1$. This implies that the steady-state interest rate satisfies:

$$\frac{\beta R}{\pi^*} = 1 + \frac{\left(1-\gamma\beta\right)\left(1-\gamma\right)}{\left(1+\Xi\right)\left(1+\varphi\right)v\gamma\pi^*} + \frac{1-\gamma\beta}{1+\Xi}\left(1-\gamma\right)\frac{Rb^*}{\gamma\pi^*\left(1-g^*\right)}$$

where we also use the fact that $\widetilde{c} = 1 - g^*$ in steady state.

Collecting terms implies that, conditional on steady-state government spending, government debt, the inflation target and steady-state velocity (which determines φ and Ξ), the value of β consistent with a desired steady state nominal interest rate can be found by setting:

$$\beta = \left[\frac{R}{\pi^*} + \frac{1-\gamma}{\left(1+\Xi\right)\left(1+\varphi\right)v\pi^*} + \frac{\left(1-\gamma\right)Rb^*}{\left(1+\Xi\right)\pi^*\left(1-g^*\right)}\right]^{-1} \times \left[1 + \frac{1-\gamma}{\left(1+\Xi\right)\left(1+\varphi\right)v\gamma\pi^*} + \frac{\left(1-\gamma\right)Rb^*}{\left(1+\Xi\right)\gamma\pi^*\left(1-g^*\right)}\right]$$

Conditional on ζ, Z is chosen to deliver a target level of steady-state real money balances. This implies that:

$$Z = \exp\left[\frac{\zeta}{v} - \ln\zeta + \ln\left(1 - R^{-1}\right)\right]$$

where steady-state velocity v is chosen based on the target real money balance level.

Finally, to calibrate the price adjustment costs, we note that log-linearising the Phillips curve gives:

$$\hat{\pi}_t = \beta\hat{\pi}_{t+1} + \frac{\eta-1}{\Phi}\hat{w}_t$$

so that the slope of the linearised Phillips curve with respect to marginal cost is $\frac{\eta-1}{\Phi}$. In a model with Calvo (1983) contracts, the slope is:

$$\frac{(1-p)(1-p\beta)}{p}$$

where p is the probability that the price is not adjusted in each quarter (see Galí, 2008). We can replicate the slope of the linearised Phillips curve for a desired value of p by setting

$$\Phi = \frac{p(\eta-1)}{(1-p)(1-p\beta)}$$

C Cash in advance variant

In this appendix, we present a cash in advance variant of the model similar to that used by Ireland (2005). Because in this variant money is held for its transactions services, we assume that the interest rate on money is zero in all periods. This allows us to consider cases in which the nominal interest rate on bonds is also equal to zero.

C.1 Overview of the differences from the baseline model

In this variant of the model we assume that households are subject to a cash in advance constraint so that purchases of consumption goods are constrained by the quantity of cash holdings that the household has access to. The government, monetary policy and firms are modelled in the same was as in the baseline variant of the model, so we do not discuss them in detail here.

The main changes to the model structure affect the timing of events within each period. Such timing assumptions are required to clarify how the cash in advance constraint limits the spending power of the household. As a result of these timing assumptions, we need to be more explicit about the tax and transfer payments made to households. It is also convenient to redefine the price of a one period bond (so that households are assumed to purchase a bond that pays one unit of money in the following period for a price equal to the reciprocal of the gross return on the bond). This renormalisation does not have any implications for the equilibrium conditions of the model, but simplifies the exposition and derivation. As noted above, we also assume that the money pays no interest (that is $R_t^M = 1, \forall t$).

C.2 Household budget constraints and timing

As in Ireland (2005), our timing protocol is based on the worker-shopper setup introduced by Lucas (1980). At the start of period t, a household born in period j receives a monetary transfer from the government, $T_{t,s}^m \geq 0$. In addition to this transfer, the household also receives income from maturing one-period bonds purchased in the previous period. Similarly, the household also carries over any money balances that were not used for consumption in the previous period.

Asset markets open at the beginning of the period and the household decides how to allocate its asset income between money and bonds. Only the amount allocated to money can be used to purchase consumption goods, so that the cash in advance constraint is written as:

$$M^p_{j,t-1} + \widetilde{B}^p_{j,t-1} + T^m_{j,t} - \frac{\widetilde{B}^p_{j,t}}{R_{t+1}} \geq P_t c_{j,t} \tag{C.1}$$

The left-hand side of (C.1) is the quantity of money held at the start of period t. As described above, the first two terms represent the stock of previously accumulated money and (matured) one-period bonds, the second term is the monetary transfer from the government and the third term is the households investment in one period bonds that will mature at the start of period $t+1$. As noted above, our notation for bond pricing is such that a bond that pays one unit of money in period $t+1$ is purchased at price R^{-1}_{t+1} in period t. For this reason we use the notation $\widetilde{B}$ to denote bond holdings, to make it clear that they are distinct from the variable B in the baseline version of the model. As in the baseline variant of the model, the rate of return between periods t and $t+1$ is determined in period t.[39]

The right hand side of (C.1) represents the consumption expenditure of the household. Consumption is carried out by the 'shopper' who splits from the 'worker' at the beginning of the period. At the end of period t, the worker and shopper reunite and pool resources. This pooling of resources gives rise to the end of period budget constraint:

$$M^p_{j,t} \leq M^p_{j,t-1} + \widetilde{B}^p_{j,t-1} + T^m_{j,t} - \frac{\widetilde{B}^p_{j,t}}{R_{t+1}} - P_t c_{j,t} + W_t n_{j,t} + D_{j,t} - T^g_{j,t}$$

which shows that the quantity of money carried forward to period $t+1$ can be no greater than the residual from the cash in advance constraint (that is, the quantity of money remaining after consumption) plus net income. Net income consists of the wage income and dividends paid to the worker by firms, net of taxes T^g_t levied by the government to finance government spending and its liabilities.

The household budget constraint can be written in real terms as:

$$m^p_{j,t} \leq \pi^{-1}_t \left(m^p_{j,t-1} + \widetilde{b}^p_{j,t-1} \right) + \tau^m_{j,t} - \frac{\widetilde{b}^p_{j,t}}{R_{t+1}} - c_{j,t} + w_t n_{j,t} + d_{j,t} - \tau^g_{j,t} \tag{C.2}$$

where lower case letters denote nominal variables deflated by the price level P_t, inflation is denoted $\pi_t \equiv P_t / P_{t-1}$ as in the main text and $\tau^x_{j,t} \equiv T^x_{j,t}/P_t, x = m, g$.

[39] The model implicitly allows households to borrow from each other using one period nominal bonds. In equilibrium, the absence of arbitrage opportunities will imply that these bonds trade at the same price as government bonds and the net supply of such bonds across all households will be zero. In fact, given the nature of the equilibrium in our model and the cash in advance constraint, young households will need to borrow (that is choose $\widetilde{B}^p_{j,t} < 0$) in order to be able to finance the optimal level of consumption in the early periods of life. A similar effect arises in the model studied by Ireland (2005).

Defining total assets as $\widetilde{a}^p_{j,t} \equiv \pi_t^{-1}\left(m^p_{j,t-1} + \widetilde{b}^p_{j,t-1}\right)$ (the household's assets at the start of period t) and assuming that the budget constraint binds allows us to write:

$$\pi_{t+1}R^{-1}_{t+1}\widetilde{a}^p_{j,t+1} = \widetilde{a}^p_{j,t} + \tau^m_{j,t} - \frac{R_{t+1}-1}{R_{t+1}}m^p_{j,t} - c_{j,t} + w_t n_{j,t} + d_{j,t} - \tau^g_{j,t}$$

which implies that

$$\begin{aligned}\widetilde{a}^p_{j,t} = &\sum_{i=0}^{\infty}\mathcal{D}_{t+i}\left[\frac{R_{t+i+1}-1}{R_{t+i+1}}m^p_{j,t+i} + c_{j,t+i} - w_{t+i}n_{j,t+i} - d_{j,t+i} + \tau^g_{j,t+i} - \tau^m_{j,t+i}\right] \\ &+ \lim_{i\to\infty}\mathcal{D}_{t+i}\pi_{t+i+1}R^{-1}_{t+i+1}\widetilde{a}^p_{j,t+i}\end{aligned}$$

where the discount factor $\mathcal{D}_{t+i}$ is defined as in the main text. Assuming that the household's no Ponzi condition holds with equality, we have $\lim_{i\to\infty}\mathcal{D}_{t+i}\widetilde{a}^p_{j,t+i} = 0$ so that the intertemporal budget constraint is:

$$\widetilde{a}^p_{j,t} = \sum_{i=0}^{\infty}\mathcal{D}_{t+i}\left[\frac{R_{t+i+1}-1}{R_{t+i+1}}m^p_{j,t+i} + c_{j,t+i} - w_{t+i}n_{j,t+i} - d_{j,t+i} + \tau^g_{j,t+i} - \tau^m_{j,t+i}\right] \tag{C.3}$$

C.3 Household optimization

Analogous to the assumptions in the baseline model, the household solves:

$$\max\sum_{t=0}^{\infty}\beta^t\vartheta_t\left[\ln c_{j,t} - \frac{\chi_{j,t}}{1+\psi}n^{1+\psi}_{j,t}\right] \tag{C.4}$$

so that preferences correspond to those in a variant of the baseline model with $\gamma = 1$.

Utility maximization subject to the cash in advance constraint and the budget constraint (C.2) gives first order conditions with respect to bonds, labor supply, consumption and money:

$$\frac{\omega_{j,t}+\xi_{j,t}}{R_{t+1}} = \beta\frac{\omega_{j,t+1}+\xi_{j,t+1}}{\pi_{t+1}} \tag{C.5}$$

$$\vartheta_t\chi_{j,t}n^{\psi}_{j,t} = \omega_{j,t}w_t \tag{C.6}$$

$$\frac{\vartheta_t}{c_{j,t}} = \omega_{j,t} + \xi_{j,t} \tag{C.7}$$

$$\omega_{j,t} = \beta\frac{\omega_{j,t+1}+\xi_{j,t+1}}{\pi_{t+1}} \tag{C.8}$$

where $\omega_{j,t}$ is the multiplier on the budget constraint and $\xi_{j,t}$ is the multiplier on the cash in advance constraint.

Under the assumption that the budget constraint binds (so that $\omega_{j,t} \neq 0, \forall t$) we can rearrange the first order conditions to give:

$$\frac{\vartheta_t}{c_{j,t}} = \beta \frac{R_{t+1}}{\pi_{t+1}} \frac{\vartheta_{t+1}}{c_{j,t+1}} \tag{C.9}$$
$$\chi \bar{c}_t n_{j,t}^{\psi} = \frac{w_t}{R_{t+1}}$$

where the second equation is the labour supply equation. Note that because the marginal value of consumption is affected by the multiplier on the cash in advance constraint, the labour supply relationship now depends on the short-term bond rate (the opportunity cost of holding cash) giving rise to a Tobin effect.

As long as the short-term bond rate is strictly positive, the cash in advance constraint binds. This means that we can combine (C.2) and (C.1) (written in real terms) to give:

$$m_{t,j}^p = w_t n_{t,j} + d_{j,t} - \tau_{j,t}^g \tag{C.10}$$

which can be written as a money demand equation by using the labour supply condition to eliminate hours worked:

$$m_{t,j}^p = w_t \left(\frac{w_t}{\chi \bar{c}_t R_{t+1}} \right)^{\frac{1}{\psi}} + d_{j,t} - \tau_{j,t}^g$$

Under the assumption that dividends are distributed equally to all households and taxes are levied equally on all households, this implies that all households will hold an identical stock of real money balances, given by:[40]

$$m_{j,t}^p = \bar{m}_t^p = w_t \left(\frac{w_t}{\chi \bar{c}_t R_{t+1}} \right)^{\frac{1}{\psi}} + \bar{d}_t - \bar{\tau}_t^g$$

which is useful for aggregation purposes.

C.4 The government budget constraint, monetary and fiscal policies

The period budget constraint of the government in real, aggregate, terms is given by:

$$m_t^g + R_{t+1}^{-1} \widetilde{b}_t^g = \pi_t^{-1} \left(m_{t-1}^g + \widetilde{b}_{t-1}^g \right) + g_t + \tau_t^m - \tau_t^g$$

We assume that monetary and fiscal policies are coordinated. There is a fiscal rule for τ_t^g that ensures that the government's solvency condition is satisfied. The short-term bond rate is set according to a monetary policy rule, subject to a lower bound:

$$R_{t+1} = \max \left\{ R \left(\frac{\pi_t}{\pi^*} \right)^{\theta_\pi} \left(\frac{y_t}{y} \right)^{\theta_y}, 1 \right\} \tag{C.11}$$

[40] Note that this implies that all households hold the same quantity of money at the *end* of the period. Differing consumption levels can be financed while satisfying the cash in advance constraint through the appropriate trading of bonds.

which is similar to the monetary policy rule for the baseline version of the model, with $\underline{R} = 1$. The only difference in the specification of the rule is that the output gap is measured relative to the steady state level of output. In the baseline model we observed that, in response to shocks to ϑ, the flexible price level of output remained at its steady state value. As described below, the presence of the cash in advance constraint generates a 'cost channel' effect in the Phillips curve which means that the flexible price allocations in this model are affected by the behaviour of monetary policy. To make the experiments as comparable as possible across the two model variants, we choose a measure of potential output that behaves identically to ϑ shocks in the two variants.

When the monetary policy rule (C.11) prescribes a strictly positive interest rate on bonds ($R_{t+1} > 1$), the cash in advance constraint binds and the monetary transfer τ_t^m is chosen to deliver the value of R_{t+1} implied by the monetary policy rule.

When the policy rule is constrained at $R_{t+1} = 1$, the cash in advance constraint does not bind and we assume that the monetary transfer τ_t^m is chosen to ensure that the total money stock satisfies:

$$\frac{M_t}{M_{t-1}} = \left(\frac{M_{t-1}}{M_{t-2}}\right)^{\rho_m} (\pi^*)^{(1-\rho_m)} \exp\left(\varepsilon_t^m\right) \tag{C.12}$$

We adopt the same fiscal policy assumptions as in the baseline model, namely that real government spending and debt are held fixed in current value terms:

$$\begin{aligned} g_t &= g^* \\ \widetilde{b}_t &= R_{t+1} b^* \end{aligned}$$

where the second equation imposes the same fiscal policy as in the baseline model.

C.5 Aggregation

As in the baseline version of the model, we need to take care over the aggregation of assets because of the alternative timing notation for total assets and the stocks of money and bonds.

To derive the consumption function for the individual household, first note that the Euler equation (C.9) implies that:

$$c_{j,t+1} = \beta \frac{R_{t+1}}{\pi_{t+1}} \frac{\vartheta_{t+1}}{\vartheta_t} c_{j,t}$$

and hence that

$$c_{j,t+i} = (\gamma\beta)^i \mathcal{D}_{t+i}^{-1} \frac{\vartheta_{t+i}}{\vartheta_t} c_{j,t} \tag{C.13}$$

where the discount factor $\mathcal{D}_{t+i}$ is defined in (B.8), identically to the baseline model.

Substituting into the household's intertemporal budget constraint (C.3) gives:

$$\begin{aligned}\widetilde{a}^{p}_{j,t} &= \sum_{i=0}^{\infty} \mathcal{D}_{t+i}\left[\frac{R_{t+i+1}-1}{R_{t+i+1}} m^{p}_{j,t+i} + (\gamma\beta)^{i}\,\mathcal{D}^{-1}_{t+i}\frac{\vartheta_{t+i}}{\vartheta_t} c_{j,t} - w_{t+i}n_{j,t+i} - d_{j,t+i} + \tau^{g}_{j,t+i} - \tau^{m}_{j,t+i}\right] \\ &= \sum_{i=0}^{\infty} \mathcal{D}_{t+i}\left[\frac{R_{t+i+1}-1}{R_{t+i+1}} m^{p}_{j,t+i} - w_{t+i}n_{j,t+i} - d_{j,t+i} + \tau^{g}_{j,t+i} - \tau^{m}_{j,t+i}\right] + \sum_{i=0}^{\infty} (\gamma\beta)^{i}\,\frac{\vartheta_{t+i}}{\vartheta_t} c_{j,t}\end{aligned}$$

which implies that

$$c_{j,t} = \mu_t\left[\widetilde{a}^{p}_{j,t} + w_{t+i}n_{j,t+i} + d_{j,t+i} + \tau^{m}_{j,t+i} - \tau^{g}_{j,t+i} - \frac{R_{t+i+1}-1}{R_{t+i+1}} m^{p}_{j,t+i}\right]$$

where the marginal propensity to consume satisfies

$$\mu_t^{-1} = \sum_{i=0}^{\infty} (\gamma\beta)^{i}\,\frac{\vartheta_{t+i}}{\vartheta_t} = 1 + \gamma\beta\frac{\vartheta_{t+1}}{\vartheta_t}\mu_{t+1}^{-1}$$

These conventions imply that aggregating the consumption function across households gives:

$$c_t = \mu_t\left[\widetilde{a}^{p}_{t} + \sum_{i=0}^{\infty} \mathcal{D}_{t+i}\left(w_{t+i}n_{t+i} + d_{t+i} + \tau^{m}_{t+i} - \tau^{g}_{t+i} - \frac{R_{t+i+1}-1}{R_{t+i+1}} m^{p}_{t+i}\right)\right]$$

which is valid because all households make identical labour supply decisions, all dividends and taxes are distributed equally across all households and (hence) all households hold the same real money balances.

C.6 Equilibrium and parsimonious model representation

The goods market clearing condition and production function equations are identical to the baseline model:

$$\begin{aligned} y_t &= c_t + g_t + \frac{\Phi}{2}\left(\frac{\pi_t}{\pi^*} - 1\right)^2 = c_t + g^* + \frac{\Phi}{2}\left(\frac{\pi_t}{\pi^*} - 1\right)^2 \\ y_t &= n_t \end{aligned}$$

The labour supply condition and production function can be combined to show that the real wage satisfies:

$$w_t = \chi y_t^{\psi} c_t R_{t+1}$$

We can use this in the pricing equation (which is identical to the baseline model), to give:

$$\frac{\Phi\pi_t}{y_t\pi^*}\left(\frac{\pi_t}{\pi^*} - 1\right) = 1 - \eta + \eta\chi y_t^{\psi} c_t R_{t+1} + \frac{\Phi}{y_t}\frac{\pi_{t+1}}{R_{t+1}}\frac{\pi_{t+1}}{\pi^*}\left(\frac{\pi_{t+1}}{\pi^*} - 1\right)$$

which implies that the cash in advance constraint gives rise to a 'cost channel' in the

Phillips curve.

Using the definition of dividends and imposing goods and asset market clearing conditions to the consumption function gives:

$$\begin{aligned} c_t &= \mu_t \left[\widetilde{a}_t + \sum_{i=0}^{\infty} \mathcal{D}_{t+i} \left(y_{t+i} - \frac{\Phi}{2} \left(\frac{\pi_{t+i}}{\pi^*} - 1 \right)^2 + \tau^m_{t+i} - \tau^g_{t+i} - \frac{R_{t+i+1} - 1}{R_{t+i+1}} m_{t+i} \right) \right] \\ &= \mu_t \left[\widetilde{a}_t + \sum_{i=0}^{\infty} \mathcal{D}_{t+i} \left(c_{t+i} + g_{t+i} + \tau^m_{t+i} - \tau^g_{t+i} - \frac{R_{t+i+1} - 1}{R_{t+i+1}} m_{t+i} \right) \right] \end{aligned}$$

so that

$$c_{t+1} = \mu_{t+1} \left[\widetilde{a}_{t+1} + \mathcal{D}^{-1}_{t+1} \sum_{i=1}^{\infty} \mathcal{D}_{t+i} \left(c_{t+i} + g_{t+i} + \tau^m_{t+i} - \tau^g_{t+i} - \frac{R_{t+i+1} - 1}{R_{t+i+1}} m_{t+i} \right) \right]$$

and hence

$$\begin{aligned} \mathcal{D}_{t+1} \frac{\mu_t}{\mu_{t+1}} c_{t+1} - c_t =& \mathcal{D}_{t+1} \mu_t \widetilde{a}_{t+1} - \mu_t \widetilde{a}_t - \mu_t \left(c_t + g_t + \tau^m_t - \tau^g_t - \frac{R_{t+1} - 1}{R_{t+1}} m_t \right) \\ =& \mathcal{D}_{t+1} \mu_t \widetilde{a}_{t+1} - \mu_t \left(c_t + \widetilde{a}_t + g_t + \tau^m_t - \tau^g_t - \frac{R_{t+1} - 1}{R_{t+1}} m_t \right) \end{aligned}$$

Evaluating the government budget constraint at asset market equilibrium gives:

$$m_t + R^{-1}_{t+1} \widetilde{b}_t = \pi^{-1}_t \left(m_{t-1} + \widetilde{b}_{t-1} \right) + g_t + \tau^m_t - \tau^g_t = \widetilde{a}_t + g_t + \tau^m_t - \tau^g_t$$

which implies that

$$\begin{aligned} \mathcal{D}_{t+1} \frac{\mu_t}{\mu_{t+1}} c_{t+1} - c_t =& \mathcal{D}_{t+1} \mu_t \widetilde{a}_{t+1} - \mu_t \left(c_t + m_t + R^{-1}_{t+1} \widetilde{b}_t - \frac{R_{t+1} - 1}{R_{t+1}} m_t \right) \\ =& \mathcal{D}_{t+1} \mu_t \pi^{-1}_{t+1} \left(\widetilde{b}_t + m_t \right) - \mu_t \left[c_t + R^{-1}_{t+1} \left(\widetilde{b}_t + m_t \right) \right] \\ =& \gamma R^{-1}_{t+1} \mu_t \left(\widetilde{b}_t + m_t \right) - \mu_t \left[c_t + R^{-1}_{t+1} \left(\widetilde{b}_t + m_t \right) \right] \\ =& (\gamma - 1) R^{-1}_{t+1} \mu_t \left(\widetilde{b}_t + m_t \right) - \mu_t c_t \end{aligned}$$

Re-arranging for c and using the definition of $\mathcal{D}_{t+1}$ gives:

$$c_t = (1 - \mu_t)^{-1} \left[\frac{\gamma \pi_{t+1}}{R_{t+1}} \frac{\mu_t}{\mu_{t+1}} c_{t+1} + (1 - \gamma) R^{-1}_{t+1} \mu_t \left(\widetilde{b}_t + m_t \right) \right]$$

To pin down equilibrium in the money market, we need to be explicit about how the fiscal policy instruments τ^g and τ^m are determined. We assume that τ^g_t is adjusted to finance government spending and the interest payments on debt required to hold the debt stock constant. That is:

$$\tau^g_t = g^* + \left(\pi^{-1}_t R_t - 1 \right) b^*$$

which implies that τ^m is used to finance money creation:

$$\tau_t^m = m_t - \pi_t^{-1} m_{t-1}$$

When the cash in advance constraint binds, we aggregate equation (C.10) and impose market clearing to give:

$$m_t = w_t n_t + d_t - \tau_t^g = y_t - \frac{\Phi}{2}\left(\frac{\pi_t}{\pi^*} - 1\right)^2 - \tau_t^g = c_t + g_t - \tau_t^g$$

and imposing the rule for τ_t^g gives:

$$m_t = c_t - \left(\pi_t^{-1} R_t - 1\right) b^*$$

We can collect the equations together into two blocks. The first block of equations hold in all periods (assuming that the household budget constraint binds):

$$\begin{aligned}
y_t &= c_t + g^* + \frac{\Phi}{2}\left(\frac{\pi_t}{\pi^*} - 1\right)^2 \\
\frac{\Phi \pi_t}{y_t \pi^*}\left(\frac{\pi_t}{\pi^*} - 1\right) &= 1 - \eta + \eta \chi y_t^{\psi} c_t R_{t+1} + \frac{\Phi}{y_t} \frac{\pi_{t+1}}{R_{t+1}} \frac{\pi_{t+1}}{\pi^*}\left(\frac{\pi_{t+1}}{\pi^*} - 1\right) \\
c_t &= (1 - \mu_t)^{-1}\left[\frac{\gamma \pi_{t+1}}{R_{t+1}} \frac{\mu_t}{\mu_{t+1}} c_{t+1} + (1 - \gamma) R_{t+1}^{-1} \mu_t \left(R_{t+1} b^* + m_t\right)\right] \\
\mu_t^{-1} &= 1 + \gamma \beta \frac{\vartheta_{t+1}}{\vartheta_t} \mu_{t+1}^{-1} \\
\Delta \ln \vartheta_t &= \rho_\vartheta \Delta \ln \vartheta_{t-1} + \varepsilon_t^\vartheta
\end{aligned}$$

which provide solutions for the sequences $\{y_t, c_t, \pi_t, \mu_t, \vartheta_t\}_{t=0}^{\infty}$, conditional on $\left\{\varepsilon_t^\vartheta\right\}_{t=0}^{\infty}$ and solutions for the sequences $\{m_t, R_{t+1}\}_{t=0}^{\infty}$.

The second block of equations can be solved for the sequence $\{m_t, R_{t+1}\}_{t=0}^{\infty}$. There are two variants, depending on whether the monetary policy rule is constrained by the zero bound. If the monetary policy rule is unconstrained by the zero bound, then the interest rate is determined by the Taylor rule. In this case, the cash in advance constraint binds and this determines real money balances:

$$\begin{aligned}
R_{t+1} &= R\left(\frac{\pi_t}{\pi^*}\right)^{\theta_\pi}\left(\frac{y_t}{y}\right)^{\theta_y} \\
m_t &= c_t - \left(\pi_t^{-1} R_t - 1\right) b^*
\end{aligned}$$

Alternatively, the short-term bond rate may be constrained by the zero lower bound, in which case the cash in advance constraint does not bind and the quantity of real money balances is determined by the rule (C.12), which we can write as:

$$R_{t+1} = 1$$
$$\frac{m_t \pi_t}{m_{t-1}} = \left(\frac{m_{t-1}\pi_{t-1}}{m_{t-2}}\right)^{\rho_m} (\pi^*)^{(1-\rho_m)} \exp\left(\varepsilon_t^m\right)$$

C.7 Steady state

As for the baseline model, government policy is considered as exogenous, so we search for a steady state conditional on $g^* \in [0,1)$, $b^* > 0$ and π^*. We consider a steady state in which the zero bound is not binding (by virtue of the inflation target being sufficiently high). In steady state, $\vartheta = 1$ and $\mu = 1 - \beta$. The monetary policy rule ensures that $\pi = \pi^*$.

The parameter χ is chosen to normalise steady-state output to unity ($y = 1$, so that $c = 1 - g^*$), which requires:

$$0 = 1 - \eta + \eta\chi\left(1 - g^*\right) R$$

or

$$\chi = \frac{\eta - 1}{\eta}\frac{1}{R\left(1 - g^*\right)}$$

In a steady state with a positive interest rate, the cash in advance constraint will bind, implying that

$$m = c - \left(\left(\pi^*\right)^{-1} R - 1\right) b^*$$
$$= 1 - g^* - \left(\left(\pi^*\right)^{-1} R - 1\right) b^*$$

As for the baseline model, our calibration approach is to calibrate β with reference to a target rate of return on short-term bonds. Treating R as a 'parameter' therefore implies that steady-state real money balances are fully determined.

In steady state, the consumption equation implies:

$$c = (\gamma\beta)^{-1}\left[\frac{\gamma\pi^*}{R}c + (1-\gamma) R^{-1}(1-\gamma\beta)(Rb^* + m)\right]$$

so that

$$\gamma\beta = \frac{\gamma\pi^*}{R} + (1-\gamma)(1-\gamma\beta)\left(\frac{b^*}{1-g^*} + \frac{m}{R(1-g^*)}\right)$$

which implies that:

$$\beta = \gamma^{-1}\left(1 + (1-\gamma)\left(\frac{b^*}{1-g^*} + \frac{m}{R(1-g^*)}\right)\right)^{-1}\left[\frac{\gamma\pi^*}{R} + (1-\gamma)\left(\frac{b^*}{1-g^*} + \frac{m}{R(1-g^*)}\right)\right]$$

D Additively separable money demand

This variant of the model abstracts from transactions frictions and assumes that household derive utility from holding real money balances. The maximization problem is:

$$\max \sum_{t=0}^{\infty} (\gamma\beta)^t \vartheta_t \left[(1-\alpha) \ln c_{j,t} + \alpha \ln \left(\frac{M_{j,t}^p}{P_t} \right) - \frac{\chi_{j,t}}{1+\psi} n_{j,t}^{1+\psi} \right] \tag{D.1}$$

subject to:

$$\frac{M_{j,t}^p}{P_t} + \frac{B_{j,t}^p}{P_t} = \gamma^{-1} \left[\frac{R_t^M M_{j,t-1}^p}{P_t} + \frac{R_t B_{j,t-1}^p}{P_t} \right] + \tilde{w}_{j,t} - c_{j,t} \tag{D.2}$$

The first order conditions deliver an Euler equation for consumption:

$$c_{j,t+1} = \beta \frac{\vartheta_{t+1}}{\vartheta_t} \frac{R_{t+1}}{\pi_{t+1}} c_{j,t} \tag{D.3}$$

and a money demand function:

$$m_{j,t}^p = \frac{\alpha}{1-\alpha} \frac{R_{t+1}}{R_{t+1} - R_{t+1}^M} c_{j,t} \tag{D.4}$$

Making the same assumptions about the properties of $\chi_{j,t}$ as the baseline model gives rise to the following labor supply relationship:

$$\chi n_{j,t}^{\xi} = (1-\alpha) \frac{w_t}{c_t} \tag{D.5}$$

As in the baseline model, we can derive the intertemporal household budget constraint:

$$a_{j,t}^p = \gamma \sum_{i=0}^{\infty} \mathcal{D}_{t+i} \left(c_{j,t+i} - w_{t+i} n_{j,t+i} - d_{j,t+i} + \tau_{j,t+i} + \frac{R_{t+i+1} - R_{t+i+1}^M}{R_{t+i+1}} m_{j,t+i}^p \right) \tag{D.6}$$

We can substitute the household's money demand equation into this gives:

$$a_{j,t}^p = \gamma \sum_{i=0}^{\infty} \mathcal{D}_{t+i} \left(c_{j,t+i} - w_{t+i} n_{j,t+i} - d_{j,t+i} + \tau_{j,t+i} + \frac{\alpha}{1-\alpha} c_{j,t+i} \right)$$

which can be rearranged to:

$$(1-\alpha)^{-1} \sum_{i=0}^{\infty} \mathcal{D}_{t+i} c_{j,t+i} = \gamma^{-1} a_{j,t}^p + \sum_{i=0}^{\infty} \mathcal{D}_{t+i} \left(w_{t+i} n_{j,t+i} + d_{j,t+i} - \tau_{j,t+i} \right) \tag{D.7}$$

The Euler equation (D.3) implies that

$$c_{j,t+i} = (\gamma\beta)^i \mathcal{D}_{t+i}^{-1} \frac{\vartheta_{t+i}}{\vartheta_t} c_{j,t} \tag{D.8}$$

Using (D.8) allows us to write (D.7) in terms of current consumption:

$$c_{j,t} = (1-\alpha)\,\mu_t \left[\gamma^{-1} a^p_{j,t} + \sum_{i=0}^{\infty} \mathcal{D}_{t+i}\left(w_{t+i} n_{j,t+i} + d_{j,t+i} - \tau_{j,t+i}\right)\right] \tag{D.9}$$

where μ is the marginal propensity to consume from wealth, given by:

$$\mu_t = \left(\sum_{i=0}^{\infty} (\gamma\beta)^i \frac{\vartheta_{t+i}}{\vartheta_t}\right)^{-1} \tag{D.10}$$

which implies that:

$$\mu_t^{-1} = 1 + \gamma\beta \frac{\vartheta_{t+1}}{\vartheta_t} \mu_{t+1}^{-1} \tag{D.11}$$

The same aggregation and manipulation of the consumption function detailed in Appendix B.2 for the baseline model deliver an aggregate consumption equation:

$$c_t = (1-\mu_t)^{-1} \frac{\gamma \pi_{t+1}}{R_{t+1}} \left[\frac{\mu_t}{\mu_{t+1}} c_{t+1} + (1-\alpha)\,\mu_t\,(1-\gamma)\,\gamma^{-1} a_{t+1}\right] \tag{D.12}$$

The aggregate pricing equation in this variant is given by:

$$\frac{\Phi \pi_t}{\pi^* y_t}\left(\frac{\pi_t}{\pi^*} - 1\right) = 1 - \eta + \eta \frac{\chi c_t y_t^{\psi}}{1-\alpha} + \Phi \frac{\pi_{t+1}}{R_{t+1}} \frac{\pi_{t+1}}{\pi^* y_t}\left(\frac{\pi_{t+1}}{\pi^*} - 1\right)$$

where the difference from the baseline model reflects the change in the labor supply relationship.

We interpret the flexible price economy as one in which there are no price adjustment costs $\Phi = 0$ and monetary policy is set to ensure that inflation is at target: $\pi_t = \pi^*, \forall t$.

Under these assumptions, the resource constraint and Phillips curve become:

$$y_t^f = c_t^f + g_t \tag{D.13}$$

$$0 = 1 - \eta + \eta \frac{\chi c_t^f \left(y_t^f\right)^{\psi}}{1-\alpha} \tag{D.14}$$

where the f superscript denotes a quantity from the flexible price economy. These two equations can be solved jointly for the levels of consumption and output that would prevail under flexible prices. It is immediate that preference shocks ϑ have no effect on flexible price output and consumption.

To ensure that consumption and output do not respond, the short-term nominal interest rate and (flexible price) money demand must adjust accordingly. The required movements can be found by (jointly) solving flexible price analogues of the consumption and money demand equations. in particular, we have:

$$\left(1-\mu_t\right)c_t^f = \frac{\gamma\pi^*}{R_{t+1}^f}\frac{\mu_t}{\mu_{t+1}}c_{t+1}^f + \left(1-\alpha\right)\mu_t\left(1-\gamma\right)\left(R_{t+1}^f\right)^{-1}\left(m_t^f + R_{t+1}^f b^*\right)$$

$$m_t^f = \frac{\alpha}{1-\alpha}\frac{R_{t+1}^f}{R_{t+1}^f - R_{t+1}^M}c_t^f$$

where we compute flexible price allocations under the assumption that the flexible price return on money is the same as the actual return on money (R_{t+1}^M) and the bond stock is held fixed.[41]

[41]These assumptions can be relaxed at the expense of increasing the size of the flexible price block of the model.