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Technical Annex to Working Paper No. 380
Evaluating and estimating a DSGE model for the United Kingdom

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1 Introduction

This technical annex provides a detailed treatment of the derivation of the model used in 'Evaluating and estimating a DSGE model for the United Kingdom'. In Section 2 we set out the structure of the model in detail. In Section 3 we solve for the steady state of the model and in Section 4 we present the linearised model.

2 Model derivation

We model a small open economy inhabited by a continuum of infinitely lived households, indexed by j along the interval [0,1], as well as firms, indexed by i along the interval [0,1]. Households own all capital in the economy and rent it out to the production sector. The total supply of capital services on the rental market is determined by the capital accumulation and utilisation decision of households. Households also supply labour services to firms and we assume that each household specialises in the supply of only one type of labour and that there are equal number of households supplying each type. Monopolistically competitive households set wages in staggered contracts with timing like that in Calvo (1983). Household preferences exhibit habit formation in consumption. Households have access to a complete set of state-contingent claims, which insure them at the domestic level, as well as risk-free nominal bonds issued by foreign governments. We assume that agents must pay fees to domestic financial intermediaries in order to hold foreign bonds. Firms are composed of *producers* and *bundlers*. Each monopolistically competitive producer produces a single differentiated intermediate good using capital and labour. 'Bundlers' combine these intermediate goods with imported intermediate goods to produce final goods which they sell domestically and abroad. Firms engage in local currency pricing (LCP) and invoice exports in foreign currency. We assume that adjusting nominal prices is costly by introducing Rotemberg (1982) type price adjustment costs.

2.1 Households

All individuals within a country have identical preferences over a real consumption index, c_t, hours worked, h_t, and real money balances, MON_t/PC_t (where MON_t denotes nominal money holdings and PC_t is the consumer price index, to be shown later. A typical household j maximises lifetime utility which is separable in its three arguments and defined as:

$$E_t \sum_{r=0}^{\infty} \beta^r \left\{ \frac{\sigma^c}{\sigma^c - 1} \left[\frac{c_{t+r}(j)}{c_{t+r-1}^{\psi^{hab}}} \right]^{\frac{\sigma_c - 1}{\sigma_c}} - (\kappa^h)^{-\frac{1}{\sigma^h}} \frac{\sigma^h}{\sigma^h + 1} [h_{t+r}(j)]^{\frac{\sigma^h + 1}{\sigma^h}} + (\kappa^{mon})^{\frac{1}{\sigma^c}} \frac{\sigma^c}{\sigma^c - 1} \left[\frac{MON_{t+r}(j)}{PC_{t+r}} \right]^{\frac{\sigma_c - 1}{\sigma_c}} \right\},$$

(1)

where E_t is the rational expectations operator, $0 < \beta < 1$ is the subjective discount factor. The parameter $\kappa^{mon} > 0$ is the utility weight of money balances in overall utility while the parameter $\kappa^h > 0$ is the weight of *dis*utility from labour supply. When the parameter $\psi^{hab} > 1$ household preferences allow for (external) habit formation in consumption. The parameter $\sigma^c > 0$ is the

elasticity of intertemporal substitution in consumption (and money balances) and the parameter $\sigma^h > 0$ is the elasticity of intertemporal substitution in labour supply.[1]

In each period, household j's expenditure on consumption and investment goods and asset accumulation must equal its disposable income:

$$PC_t c_t(j) + PI_t I_t(j) + MON_t(j) - MON_{t-1}(j) + E_t \rho_{t+1,t} B_t(j) - B_{t-1}(j) + \frac{BF_t(j)}{ER_t}$$

$$-(1 + rf_{t-1}) \frac{BF_{t-1}(j)}{ER_t} = W_t h_t(j) + R_t k_t^s(j) + \int_0^1 \Pi_t^v(j,i) di - PC_t \tau_t$$

$$-PC_t \frac{\chi^{bf}}{2} \left(\frac{BF_t(j)}{PC_t ER_t} - nfa^{ss} \right)^2 + PC_t \Gamma_t. \tag{2}$$

Asset accumulation consists of increases in money holdings, increases in acquisition of domestic state-contingent claims, and increases in foreign asset holdings. $B_t(j)$ denotes the quantity of state-contingent claims purchased by the household in period t to be delivered in each state of the subsequent period. $\rho_{t+1,t}$ represents the period t price of a state-contingent claim that will pay one unit of currency in a particular state of nature in period $t+1$ divided by the probability of occurrence of that state given information available in period t. $B_{t-1}(j)$ is the value of the household's claims given the current realisation of the state of nature. $BF_t(j)$ stands for the value of foreign nominal bonds acquired by the household in period t and ER_t is the time-t nominal exchange rate. An increase in ER_t reflects an *appreciation* of the domestic currency. rf_{t-1} is the nominal rate of interest on holdings of foreign assets between t-1 and t and $BF_{t-1}(j)$ denotes the value of such assets purchased by the household in period t-1. The remaining terms in the budget constraint are household's consumption expenditure, $PC_t c_t(j)$, purchase of investment goods, $PI_t I_t$, labour income, $W_t h_t(j)$, rental income from the supply of capital services to firms, $R_t k_t^s(j)$, aggregate profits received from firms, where $\Pi_t^v(j,i)$ denotes the profit of firm i received by household j, total lump-sum tax/transfer payments, $PC_t \tau_t(j)$, which in equilibrium are equal to government spending, g_t, less the seignorage revenue of the government,[2] the per-capita cost of holding foreign bonds in excess of the steady-state value, $PC_t \frac{\chi^{bf}}{2} \left(\frac{BF_t(j)}{PC_t ER_t} - nfa^{ss} \right)^2$, and finally the rebate given to the household by the financial intermediary, $PC_t \Gamma_t$, which in equilibrium is equal to the cost of holding foreign bonds. Following Ghironi and Melitz (2005) we assume that agents must pay fees to domestic financial intermediaries in order to hold foreign bonds. These fees are quadratic in the deviation of the real stock of bonds from their steady-state value. Financial intermediaries are then assumed to rebate the revenues from bond-holding fees to domestic households. Here, international assets markets are incomplete as only nominal government bonds can be traded across countries. As argued by Ghironi and Melitz (2005), in the absence of the cost of holding bonds, the incomplete market assumption would imply that the steady-state net foreign assets would be indeterminate and that the model would be non-stationarity. Adding the cost of holding bonds pins down the steady state and ensures mean reversion in the long run.

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[1] Notice that σ^c is also the inverse of the coefficient of relative risk aversion and σ^h is the labour supply elasticity.

[2] Since Ricardian equivalence holds in our set-up we can assume that government runs a balanced budget each period.

2.1.1 The composite consumption good and demands

The composite consumption index of household j is a standard constant elasticity of substitution (CES) function of domestic and foreign consumption subindices:

$$c_t(j) \equiv \kappa^c [(1 - \psi^m)(ch_t(j))^{\frac{\sigma^m - 1}{\sigma^m}} + \psi^m (cm_t(j))^{\frac{\sigma^m - 1}{\sigma^m}}]^{\frac{\sigma^m}{\sigma^m - 1}}, \qquad (3)$$

where $ch_t(j)$ is an index of consumption of domestic goods, $cm_t(j)$ is an index of consumption of imported goods, and κ^c is a parameter. The parameter $0 < \psi^m < 1$ represents the expenditure weight of imported consumption goods in the aggregate consumption index and is (inversely) related to the degree of Home bias in preferences. The parameter $\sigma^m > 1$ is the elasticity of substitution between domestic and foreign final consumption goods. The price index associated with the minimum expenditure needed to purchase a unit of the composite consumption index given the price indices of domestic and imported goods is then:

$$PC_t \equiv \frac{1}{\kappa^c} [(1 - \psi^m)^{\sigma^m} (PH_t)^{1 - \sigma^m} + (\psi^m)^{\sigma^m} (PM_t)^{1 - \sigma^m}]^{\frac{1}{1 - \sigma^m}}, \qquad (4)$$

where PH_t is price index for goods produced at Home and PM_t is the price index for imported goods, both expressed in domestic currency. We choose our numeraire to be the composite consumption index and we define relative prices in relation to PC_t. That is for any price PX_t, the relative price, denoted by px_t, is given by PX_t/PC_t. Then, (4) can be rewritten as:

$$1 = \frac{1}{\kappa^c} [(1 - \psi^m)^{\sigma^m} (ph_t)^{1 - \sigma^m} + (\psi^m)^{\sigma^m} (pm_t)^{1 - \sigma^m}]^{\frac{1}{1 - \sigma^m}}. \qquad (5)$$

Consumption and price indices of domestic goods. Household j's domestic consumption index, which aggregates over all available goods (or brands), and the corresponding domestic price are given by the following CES functions:

$$ch_t(j) \equiv [\int_0^1 (ch_t(j, i))^{\frac{\sigma^{hb} - 1}{\sigma^{hb}}} di]^{\frac{\sigma^{hb}}{\sigma^{hb} - 1}} \qquad (6)$$

$$ph_t \equiv [\int_0^1 (ph_t(i))^{1 - \sigma^{hb}} di]^{\frac{1}{1 - \sigma^{hb}}}, \qquad (7)$$

where $ch_t(j, i)$ denotes the consumption of household j of brand i produced domestically and the parameter $\sigma^{hb} > 1$ is the elasticity of substitution among domestically produced brands.

Consumption and price indices of imported goods. Household j's import consumption index, which aggregates over all available foreign brands according to an analogous CES function, is given by:

$$cm_t(j) \equiv [\int_0^1 (cm_t(j, i))^{\frac{\sigma^{mb} - 1}{\sigma^{mb}}} di]^{\frac{\sigma^{mb}}{\sigma^{mb} - 1}}, \qquad (8)$$

where $cm_t(j, i)$ is consumption by household j of an imported good i and the parameter $\sigma^{mb} > 1$ is the elasticity of substitution among imported brands. And the corresponding import price index is:

$$pm_t \equiv [\int_0^1 (pm_t(i))^{1 - \sigma^{mb}} di]^{\frac{1}{1 - \sigma^{mb}}}, \qquad (9)$$

where $pm_t(i)$ is the domestic price of an imported good i.

Demand for domestic and imported goods. The optimal allocation of expenditure across domestic and imported brands yields the following demand functions:

$$ch_t(j,i) = (\frac{ph_t(i)}{ph_t})^{-\sigma^{hb}} ch_t(j) \tag{10}$$

$$cm_t(j,i) = (\frac{pm_t(i)}{pm_t})^{-\sigma^{mb}} cm_t(j). \tag{11}$$

Using equations **(10)** and **(11)** one can show that the total expenditure of household j on domestic goods consumption is $\int_0^1 ph_t(i)ch_t(j,i)di = ph_t ch_t(j)$ and on imported good consumption it is $\int_0^1 pm_t(i)cm_t(j,i)di = pm_t cm_t(j)$.

Finally, the optimal allocation of expenditure between domestic and imported goods implies:

$$ch_t(j) = (1 - \psi^m)^{\sigma^m} (ph_t)^{-\sigma^m} \frac{c_t(j)}{(\kappa^c)^{1-\sigma^m}} \tag{12}$$

$$cm_t(j) = (\psi^m)^{\sigma^m} (pm_t)^{-\sigma^m} \frac{c_t(j)}{(\kappa^c)^{1-\sigma^m}}. \tag{13}$$

The total consumption expenditure of household j is given by $c_t(j) = pm_t cm_t(j) + ph_t ch_t(j)$.

2.1.2   Capital utilisation and accumulation

Each firm i rents capitals services, $k_t^s(j)$, from household j at the real rental rate r_t. The capital services supplied by household j, which depends on utilisation decision of households and accumulated capital, is given by:

$$k_t^s(j) = z_t(j)k_{t-1}(j), \tag{14}$$

where $z_t(j)$ denotes the utilisation rate of capital by household j and $k_{t-1}(j)$ is the stock of physical capital previously accumulated by the household. As the equation shows, the capital used in the current period must be installed in the previous period but the household, in the short run, can choose the intensity with which it is used. However, setting a higher utilisation rate incurs real costs due to increased wear and tear. This utilisation cost is defined below:

$$\Delta_t^z(j) \equiv \frac{\chi^z}{1 + \sigma^z}[(z_t(j))^{1+\sigma^z} - 1]k_{t-1}(j), \tag{15}$$

where we set the steady-state utilisation rate to 1 (see Christiano, Eichenbaum and Evans (2005)). This implies that in the long run only increases in physical capital can generate more output.

The physical capital stock the household owns evolves over time according to the following equation:

$$k_t(j) = I_t(j) + (1 - \delta)k_{t-1}(j) - \Delta_t^z(j) - \Delta_t^k(j), \tag{16}$$

where $0 < \delta < 1$ is the depreciation rate of capital stock and $\Delta_t^k(j)$ is the real cost associated with a change in capital stock and is defined as:

$$\Delta_t^k(j) \equiv \frac{\chi^k}{2} \frac{[k_t(j) - (\frac{k_{t-1}}{k_{t-2}})^{\epsilon^k} k_{t-1}(j)]^2}{k_{t-1}}, \tag{17}$$

Equation (16) tells us that today's investment will increase the capital stock carried over to the next period, while any costs associated with changing the utilisation rate and level of capital stock will decrease the capital stock carried over to the next period.

Composite investment good. Household j can augment its existing physical capital stock by purchasing a composite investment good, $I_t(j)$. For simplicity, we assume that there are no imported investment goods and that composite investment good is produced in the same way as the domestic consumption good. The index of investment goods consumed by household j aggregates over all domestic brands according to an analogous CES function:

$$I_t(j) = [\int_0^1 (I_t(j,i))^{\frac{\sigma^{hb}-1}{\sigma^{hb}}} di]^{\frac{\sigma^{hb}}{\sigma^{hb}-1}}. \tag{18}$$

So the unit price of the composite investment good and the unit price of the composite consumption good will be the same ($PI_t = PH_t$) since they are both produced using the same technology. The optimal allocation of investment expenditure across Home will yield an analogous demand function:

$$I_t(j,i) = (\frac{ph_t(i)}{ph_t})^{-\sigma^{hb}} I_t(j), \tag{19}$$

where $I_t(j,i)$ is the consumption by household j of good i for investment purposes.

2.1.3 Optimal wage-setting

Following Erceg, Henderson and Levin (2000), we assume that each household j supplies a differentiated labour input to the production sector. The labour index, which aggregates over all available labour type, has the CES form:

$$h_t \equiv [\int_0^1 (h_t(j))^{\frac{\sigma^w-1}{\sigma^w}} dj]^{\frac{\sigma^w}{\sigma^w-1}}, \tag{20}$$

where the parameter $\sigma^w > 1$ is the elasticity of substitution among labour services. The aggregate nominal wage index associated with the minimum cost needed to hire a unit of the composite labour given each household's wage rate is given by:

$$W_t \equiv [\int_0^1 (W_t(j))^{1-\sigma^w} dj]^{1-\sigma^w}. \tag{21}$$

Hence, each household j faces a downward-sloping demand curve for its own labour.

$$h_t(j) = [\frac{W_t(j)}{W_t}]^{-\sigma^w} h_t. \tag{22}$$

Households set (nominal) wages in staggered contracts as outlined in Calvo (1983). In particular, a constant fraction of households (ψ^w) can renegotiate their wage contracts in each period. The

remaining fraction continues to supply labour services at the old wage rate. The average duration of wage contracts is $1/\psi^w$ quarters. Whenever a household j has not reset its contract wage since period t, then its wage rate in period $t + r$ is adjusted by an indexation factor, $\xi^w_{t,t+r}$. That is $W_{t,t+r}(j) = \xi^w_{t,t+r}\widetilde{W}_t(j)$. Here, $\widetilde{W}_t(j)$ denotes the optimal wage rate set in period t and $W_{t,t+r}(j)$ denotes the wage rate in period $t + r$ faced by a household that has set its wage rate at time t. The expression for the indexation factor is given below.

$$
\xi^w_{t,t+r} = \left\{
\begin{array}{ll}
1 & \text{if } r = 0 \\
(1 + \dot{p}^{ss})^{1-\epsilon^w}\left(\frac{W_{t+r-1}}{W_{t+r-2}}\right)^{\epsilon^w}\xi^w_{t,t+r-1} & \text{if } r \geq 1.
\end{array}
\right.
$$

This expression implies that if a household who has set wages in period t does not receive a signal to update its wage at time $t + r$ its wage rate is increased in proportion with the weighted average of the steady-state rate of (gross) inflation $1 + \dot{p}^{ss}$ and the lagged (gross) nominal wage inflation. The parameter $0 < \epsilon^w < 1$ is the weight attached to the latter. In any period t in which household j is able to reset its contract wage, $W_t(j)$, it aims to maximise the following expression:

$$
E_t \sum_{r=0}^{\infty} \beta^r (1 - \psi^w)^r \{\lambda_{t+r}\xi^w_{t,t+r}W_t(j)h_{t+r}(j) - (\kappa^h)^{-\frac{1}{\sigma^h}}\frac{\sigma^h}{\sigma^h + 1}[h_{t+r}(j)]^{\frac{\sigma^h+1}{\sigma^h}}\}, \qquad \textbf{(23)}
$$

where λ is the marginal utility of wealth (or the Lagrange multiplier for household j's budget constraint). Notice that λ does not depend on j implying that households are homogeneous with respect to wealth despite the fact that they work different hours and earn different wage rates. This is due to the presence of state-contingent securities which ensures that, in equilibrium, households are perfectly insured against income risk and therefore they are homogenous with respect to wealth although they are heterogenous with respect to hours they work and wages they earn (see Erceg *et al* (2000)). According to **(23)**, the jth household aims to maximise the discounted marginal utility from an additional unit of labour relative to its discounted marginal *dis*utility, in expected value terms assuming that it cannot reset wages again along the path. The probability that this happens for r periods is given by $(1 - \psi^w)^r$. This problem has the following first-order condition:

$$
0 = \quad E_t \sum_{r=0}^{\infty} \beta^r (1 - \psi^w)^r \lambda_{t+r}\xi^w_{t,t+r}(1 - \sigma^w)\left(\frac{\xi^w_{t,t+r}\widetilde{W}_t(j)}{W_{t+r}}\right)^{-\sigma^w}h_{t+r}
$$

$$
+ E_t \sum_{r=0}^{\infty} \beta^r (1 - \psi^w)^r (\kappa^h)^{-\frac{1}{\sigma^h}}\sigma^w\left[\left(\frac{\xi^w_{t,t+r}\widetilde{W}_t(j)}{W_{t+r}}\right)^{-\sigma^w}h_{t+r}\right]^{\frac{\sigma^h+1}{\sigma^h}}\widetilde{W}_t(j)^{-1}.
$$

We can rewrite the above first-order condition in terms of household's wage rate relative to the average wage index as:

$$
0 = \quad E_t \sum_{r=0}^{\infty} \beta^r (1 - \psi^w)^r \lambda_{t+r}\xi^w_{t,t+r}(1 - \sigma^w)\left(\frac{\xi^w_{t,t+r}W_t}{W_{t+r}}\right)^{-\sigma^w}\left(\frac{\widetilde{W}_t(j)}{W_t}\right)^{-\sigma^w}h_{t+r}
$$

$$
+ E_t \sum_{r=0}^{\infty} \beta^r (1 - \psi^w)^r (\kappa^h)^{-\frac{1}{\sigma^h}}\sigma^w\left[\left(\frac{\xi^w_{t,t+r}W_t}{W_{t+r}}\right)^{-\sigma^w}h_{t+r}\right]^{\frac{\sigma^h+1}{\sigma^h}}\left(\frac{\widetilde{W}_t(j)}{W_t}\right)^{-\sigma^w\frac{\sigma^h+1}{\sigma^h}}\widetilde{W}_t(j)^{-1}.
$$

As households that reset wages in period t are identical, in that they face the same decision problem, the optimal wage rate set at time t will be the same for all of them. Then, the optimal

nominal wage rate, $\widetilde{W}_t$, the household can renegotiate is given by:

$$\widetilde{W}_t^{1+\frac{\sigma^w}{\sigma^h}} = W_t^{\frac{\sigma^w}{\sigma^h}} \frac{(\kappa^h)^{-\frac{1}{\sigma^h}} \sigma^w}{\sigma^w - 1} \frac{E_t \sum_{r=0}^{\infty} \beta^r (1-\psi^w)^r [(\frac{\xi_{t,t+r}^w W_t}{W_{t+r}})^{-\sigma^w} h_{t+r}]^{\frac{\sigma^h+1}{\sigma^h}}}{E_t \sum_{r=0}^{\infty} \beta^r (1-\psi^w)^r \lambda_{t+r} \xi_{t,t+r}^w (\frac{\xi_{t,t+r}^w W_t}{W_{t+r}})^{-\sigma^w} h_{t+r}}. \qquad \textbf{(24)}$$

After dividing both sides of the equation by PC_t, the optimal real wage rate $\widetilde{w}_t$ is given by:

$$\widetilde{w}_t^{1+\frac{\sigma^w}{\sigma^h}} = w_t^{\frac{\sigma^w}{\sigma^h}} \frac{(\kappa^h)^{-\frac{1}{\sigma^h}} \sigma^w}{\sigma^w - 1} \frac{\Xi_t^w}{\Theta_t^w}, \qquad \textbf{(25)}$$

where

$$\Xi_t^w \equiv E_t \sum_{r=0}^{\infty} \beta^r (1-\psi^w)^r [(\frac{\xi_{t,t+r}^w W_t}{W_{t+r}})^{-\sigma^w} h_{t+r}]^{\frac{\sigma^h+1}{\sigma^h}}$$

$$\Theta_t^w \equiv E_t \sum_{r=0}^{\infty} \beta^r (1-\psi^w)^r PC_t \lambda_{t+r} \xi_{t,t+r}^w (\frac{\xi_{t,t+r}^w W_t}{W_{t+r}})^{-\sigma^w} h_{t+r}.$$

The above expression can be simplified further by noting that $\xi_{t,t+r}^w = \xi_{t,t+1}^w \xi_{t+1,t+r}^w$ as follows:

$$\Xi_t^w = h_t^{\frac{\sigma^h+1}{\sigma^h}} + \beta(1-\psi^w) E_t [\frac{\xi_{t,t+1}^w W_t}{W_{t+1}}]^{-\sigma^w(\frac{\sigma^h+1}{\sigma^h})} \Xi_{t+1}^w$$

$$= h_t^{\frac{\sigma^h+1}{\sigma^h}} + \beta(1-\psi^w) E_t [\frac{\xi_{t,t+1}^w}{1+\dot{w}_{t+1}}]^{-\sigma^w(\frac{\sigma^h+1}{\sigma^h})} \Xi_{t+1}^w$$

where

$$\xi_{t,t+1}^w = (1+\dot{p}^{ss})^{1-\epsilon^w} (\frac{W_t}{W_{t-1}})^{\epsilon^w} = (1+\dot{p}^{ss})^{1-\epsilon^w} (1+\dot{w}_t)^{\epsilon^w},$$

where $1+\dot{p}_t \equiv \frac{PC_t}{PC_{t-1}}$ denotes the rate of (gross) inflation in period t. Likewise, $1+\dot{w}_t \equiv \frac{W_t}{W_{t-1}} = (1+\dot{p}_t)\frac{w_t}{w_{t-1}}$ denotes the rate of (gross) nominal wage inflation at Home in period t. Similarly, we can derive the following expression for Θ_t^w. Here, we also make use of the definition $\Lambda_{t+r} \equiv \lambda_{t+r} PC_{t+r} = U_{c,t+r}$, where Λ_{t+r} is the real-valued Lagrange multiplier in front of household j's budget constraint (discussed further below). Then, we have

$$\Theta_t^w = U_{c,t} h_t + E_t \sum_{r=1}^{\infty} \beta^r (1-\psi^w)^r \prod_{j=1}^{r} (1+\dot{p}_{t+j})^{-1} U_{c,t+r} \xi_{t,t+r}^w (\frac{\xi_{t,t+r}^w W_t}{W_{t+r}})^{-\sigma^w} h_{t+r}$$

$$= U_{c,t} h_t + \beta(1-\psi^w) E_t (1+\dot{p}_{t+1})^{-1} \xi_{t,t+1}^w [\frac{\xi_{t,t+1}^w}{1+\dot{w}_{t+1}}]^{-\sigma^w} \Theta_{t+1}^w.$$

Notice that when prices are flexible, ie $\psi^w = 1$, then **(25)** collapses to $\widetilde{w}_t = \frac{\sigma^w}{\sigma^w-1} (\kappa^h)^{-\frac{1}{\sigma^h}} h_t^{\frac{1}{\sigma^h}} U_{c,t}^{-1}$. This is the familiar labour/leisure relationship where monopolistically competitive households set wages as a mark-up, $\frac{\sigma^w}{\sigma^w-1}$, over marginal rate of substitution of consumption for leisure, $\frac{U_{h,t}}{U_{c,t}}$ (where $U_{h,t}$ and $U_{c,t}$ denote the marginal utility of leisure and consumption, respectively).[3]

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[3] $U_{h,t} = (\kappa^h)^{-\frac{1}{\sigma^h}} h_t^{\frac{1}{\sigma^h}}$ and $U_{c,t} = \frac{1}{c_{t-1}^{\psi^{hab}}} [\frac{c_t(j)}{c_{t-1}^{\psi^{hab}}}]^{\frac{-1}{\sigma^c}}$.

The nominal wage index in period t satisfies:

$$
\begin{aligned}
W_t^{1-\sigma^w} & \equiv \int_0^1 (W_t(j))^{1-\sigma^w} dj \\
& = (1-\psi^w)\int_0^1 (\xi_{t-1,t}^w W_{t-1}(j))^{1-\sigma^w} dj + \psi^w (\widetilde{W}_t)^{1-\sigma^w} \\
& = (1-\psi^w)(\xi_{t-1,t}^w W_{t-1})^{1-\sigma^w} + \psi^w (\widetilde{W}_t)^{1-\sigma^w}.
\end{aligned}
\tag{26}
$$

where

$$
\xi_{t-1,t}^w = (1+\dot{p}^{ss})^{1-\epsilon^w}(\frac{W_{t-1}}{W_{t-2}})^{\epsilon^w} = (1+\dot{p}^{ss})^{1-\epsilon^w}(1+\dot{w}_{t-1})^{\epsilon^w}.
$$

The real wage index can then be written as:

$$
w_t^{1-\sigma^w} = (1-\psi^w)(\xi_{t-1,t}^w \frac{w_{t-1}}{1+\dot{p}_t})^{1-\sigma^w} + \psi^w (\widetilde{w}_t)^{1-\sigma^w}.
\tag{27}
$$

Equations **(26)** and **(27)** tell us that the aggregate wage index (nominal and real) at time t is a weighted average of newly set wages and the lagged wage index.

2.1.4   The household optimisation problem

A typical household j chooses $c_t(j)$, $B_t(j)$, $BF_t(j)$, $MON_t(j)$, $I_t(j)$, $W_t(j)$, $k_t(j)$, $z_t(j)$, to maximise its utility **(1)** subject to its budget constraint **(2)**, its labour demand **(22)**, and its capital accumulation **(16)**, for $t = 0, 1, ..., \infty$. Below we present the first-order conditions from this optimisation problem with the exception of the first-order condition with respect to $W_t(j)$ which we have shown in Section 2.1.3.

The first-order conditions with respect to $c_t(j)$, $B_t(j)$, $BF_t(j)$, $MON_t(j)$ yield:

$$
PC_t \lambda_t = U_{c,t}(j)
\tag{28}
$$

$$
E_t[\beta \frac{\lambda_{t+1}}{\lambda_t}] = E_t \rho_{t+1,t} \equiv \frac{1}{1+rg_t}
\tag{29}
$$

$$
E_t[\beta \frac{\lambda_{t+1}}{\lambda_t} \frac{ER_t}{ER_{t+1}}(1+rf_t)] = 1 + \chi^{bf}(\frac{BF_t(j)}{ER_t PC_t} - nfa^{ss})
\tag{30}
$$

$$
\lambda_t - E_t \beta \lambda_{t+1} = U_{MON,t}(j).
\tag{31}
$$

λ is the marginal utility of wealth (or the Lagrange multiplier for household j's budget constraint) as before. The variable rg_t is the nominal rate of return on a bond purchased for one unit of domestic currency at date t. The effective price of the bond at date t ($= \frac{1}{1+rg_t}$) is equal to the expected period t price of a state-contingent claim that will pay one unit of domestic currency in period $t+1$. The expressions for $U_{c,t}$, the marginal utility of consumption, and $U_{MON,t}$, the

marginal utility of holding money balances, are given below:

$$U_{c,t}(j) \;=\; \frac{1}{c_{t-1}^{\psi^{hab}}}\Big[\frac{c_t(j)}{c_{t-1}^{\psi^{hab}}}\Big]^{\frac{-1}{\sigma^c}}$$

$$U_{MON,t}(j) \;=\; PC_t^{\frac{1-\sigma^c}{\sigma^c}}\Big[\frac{MON_t(j)}{\kappa^{mon}}\Big]^{\frac{-1}{\sigma^c}}.$$

Let $\Lambda_t \equiv PC_t\lambda_t$ be the real valued Lagrange multiplier. Notice that we introduced this notation first in section 2.1.3. Then, **(29)** implies that:

$$E_t\Big[\beta\frac{\Lambda_{t+1}}{\Lambda_t}\Big] \cong E_t\Big[\frac{1+\dot{p}_{t+1}}{1+rg_t}\Big] \equiv \frac{1}{1+rrg_t}, \qquad \textbf{(32)}$$

where $1+\dot{p}_{t+1} = \frac{PC_{t+1}}{PC_t}$ denotes the rate of (gross) inflation at time $t+1$.[4] In deriving **(32)** we make use of the familiar Fisher parity condition to link rg_t and rrg_t, where the latter is the real rate of return on a bond that pays one unit of consumption under every state of nature at time $t+1$. The first-order conditions **(28)** and **(32)** imply the familiar consumption Euler equation, which links the marginal cost of foregoing a unit of current consumption to the expected marginal benefit in the subsequent period:

$$U_{c,t}(j) = \beta E_t(1+rrg_t)U_{c,t+1}(j). \qquad \textbf{(33)}$$

Notice that combining first-order conditions **(29)** and **(30)** yields:

$$\frac{1+rf_t}{1+rg_t} \cong E_t\frac{ER_{t+1}}{ER_t}\Big[1+\chi^{bf}\Big(\frac{BF_t(j)}{ER_tPC_t}-nfa^{ss}\Big)\Big]. \qquad \textbf{(34)}$$

Together with the Fisher parity the above equation implies that:

$$\frac{1+rrf_t}{1+rrg_t} \cong E_t\frac{q_{t+1}}{q_t}\Big[1+\chi^{bf}\Big(\frac{bf_t(j)}{q_t}-nfa^{ss}\Big)\Big], \qquad \textbf{(35)}$$

where $q_t \equiv \frac{ER_tPC_t}{PCF_t}$ is the consumption-based real exchange (the price of Home consumption in terms of foreign consumption) with PCF_t denoting the foreign consumer price index. An increase in q_t corresponds to a real *appreciation*, implying domestic goods are becoming relatively more expensive. The variable $bf_t(j) = \frac{BF_t(j)}{PCF_t}$ is the holdings of real foreign bonds acquired in period t and $(1+rrf_t)$ is the real return in the rest of the world. Equations **(34)** and **(35)** are the modified versions of the nominal and real uncovered interest rate parity (UIP) conditions which say that, for agents to be indifferent between domestic and foreign bonds, the nominal and real interest rate differentials must be equal to the nominal and real depreciation adjusted for the cost associated with holding bonds. When $\chi^{bf}=0$, equations **(34)** and **(35)** collapse to the standard nominal and real UIP conditions.

The first-order condition **(31)** can be rewritten as:

$$U_{c,t}(j) = PC_tU_{MON,t}(j) + \beta E_{t+1}\Big[\frac{U_{c,t+1}(j)}{(1+\dot{p}_{t+1})}\Big].$$

This condition states that agents must be indifferent between consuming a unit of consumption good at time-t or using the same amount to raise cash balances, enjoying the utility from money

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[4]The approximation we use in expression **(32)** will not matter for our analysis since we solve a log linearised version of the model.

holdings at period-t, and then converting the cash balances back to consumption in period $t+1$. Equation (31) can further be arranged to yield a demand curve for real money balances of the form:

$$mon_t(j) = \frac{MON_t(j)}{PC_t} = \kappa^{mon}[U_{c,t}(j)(\frac{rg_t}{1+rg_t})]^{-\sigma^c}. \tag{36}$$

Hence, the demand for real money balances decreases with an increase in the short-term nominal interest rate but increases with an increase in consumption.

The first-order condition with respect to $I_t(j)$, $k_t(j)$, and $z_t(j)$ yields:

$$\mu_t(j) = ph_t\Lambda_t \tag{37}$$

$$ph_t(1 + \frac{\partial\Delta^k_t(j)}{\partial k_t(j)}) = \beta E_t\frac{\Lambda_{t+1}}{\Lambda_t}\{r_{t+1}z_{t+1}(j) + ph_{t+1}(1 - \delta - \frac{\partial\Delta^z_{t+1}(j)}{\partial k_t(j)} - \frac{\partial\Delta^k_{t+1}(j)}{\partial k_t(j)})\} \tag{38}$$

$$r_t = ph_t\chi^z(z_t(j))^{\sigma^z}, \tag{39}$$

where

$$\frac{\partial\Delta^k_t(j)}{\partial k_t(j)} = \chi^k\frac{k_t(j) - (\frac{k_{t-1}}{k_{t-2}})^{\epsilon^k}k_{t-1}(j)}{k_{t-1}}$$

$$E_t\frac{\partial\Delta^k_{t+1}(j)}{\partial k_t(j)} = E_t[-\chi^k(\frac{k_t}{k_{t-1}})^{\epsilon^k}\frac{k_{t+1}(j) - (\frac{k_t}{k_{t-1}})^{\epsilon^k}k_t(j)}{k_t}]$$

$$E_t\frac{\partial\Delta^z_{t+1}(j)}{\partial k_t(j)} = E_t[\chi^z\frac{(z_{t+1}(j))^{1+\sigma^z} - 1}{1+\sigma^z}]$$

The variable μ is the Lagrange multiplier for the household's capital accumulation equation. Equations (37) and (38) determine the optimal path for the capital. Notice that equation (37) is the equivalent of Tobin's q: households increase their holdings of capital stock until the value of a unit of capital, $\mu_t(j)$, equals to the replacement cost of capital, $ph_t\Lambda_t$. Equation (38) states that the value of unit of capital depends on expected future return as captured by the expected rental rate times the expected rate of capital utilisation and expected future value taking into account the depreciation rate and costs associated with capital utilisation and adjustment. Equation (39) specifies the optimal rate of capital utilisation and states that the cost of increasing the utilisation rate (cost of output) should equal the real rental price of capital services. Variable capital utilisation smooths the response of the rental rate of capital, and hence the marginal cost, to fluctuations in output.

As noted previously, since every household is perfectly insured against income risk at the domestic level, the marginal utility of wealth is identical across households for all periods: $\lambda_t(j) = \lambda_t, \forall t$. And since every household faces the same prices the first-order condition (37) then implies $\mu_t(j) = \mu_t, \forall t$. Further, since preferences are identical across all households, consumption, investment, money and foreign bond holdings are also identical across all households at all times: $c_t(j) = c_t$, $I_t(j) = I_t$, $BF_t(j) = BF_t$, $MON_t(j) = MON_t, \forall t$. In addition: $z_t(j) = z_t$ and $k^s_t(j) = k^s_t, \forall t$.

2.2 Firms

All firms engage in LCP and invoice goods in the currency of the destination market. We assume that firms face different (price) elasticities of demand for their products sold in domestic and export markets. Hence, firms can price discriminate across markets.

Producers. There is a continuum of monopolistically competitive producers each supplying a single differentiated intermediate good using capital and labour only. These inputs are then used by bundlers in the production of final consumption goods which are then supplied to domestic and export markets. We denote the output of a typical producer i used in the production of final domestic goods by $y_t^{hv}(i)$ and that used in the production of final export goods by $y_t^{xv}(i)$. The value added of producer i, $y_t^v(i) = y_t^{hv}(i) + y_t^{xv}(i)$, is given by a CES technology:

$$y_t^v(i) = tfp_t[(1-\alpha)(h_t(i))^{\frac{\sigma^y-1}{\sigma^y}} + \alpha(k_t^s(i))^{\frac{\sigma^y-1}{\sigma^y}}]^{\frac{\sigma^y}{\sigma^y-1}}. \tag{40}$$

tfp_t is the economy-wide productivity level and $\sigma^y > 0$ is the elasticity of substitution between capital and labour. The variable $k_t^s(i) = \int_0^1 k_t^s(j)dj$ is a measure of total capital services rented by producer i from households and $h_t(i)$ is the hours of the aggregate labour index shown in **(20)** hired by producer i from households. Solving the cost minimisation problem of the producer yields the optimal relationship between capital and labour, the conditional factor demand functions, and the marginal cost function of the producer:[5]

$$\frac{w_t}{r_t} = \frac{1-\alpha}{\alpha}[\frac{k_t^s(i)}{h_t(i)}]^{\frac{1}{\sigma^y}} \tag{41}$$

$$h_t(i) = (\frac{w_t}{1-\alpha})^{-\sigma^y}[(1-\alpha)^{\sigma^y}w_t^{1-\sigma^y} + \alpha^{\sigma^y}r_t^{1-\sigma^y}]^{\frac{\sigma^y}{1-\sigma^y}}\frac{y_t^v(i)}{tfp_t} \tag{42}$$

$$k_t^s(i) = (\frac{r_t}{\alpha})^{-\sigma^y}[(1-\alpha)^{\sigma^y}w_t^{1-\sigma^y} + \alpha^{\sigma^y}r_t^{1-\sigma^y}]^{\frac{\sigma^y}{1-\sigma^y}}\frac{y_t^v(i)}{tfp_t} \tag{43}$$

$$mc_t = [(1-\alpha)^{\sigma^y}w_t^{1-\sigma^y} + \alpha^{\sigma^y}r_t^{1-\sigma^y}]^{\frac{1}{1-\sigma^y}}\frac{1}{tfp_t}. \tag{44}$$

Notice that producers share the same marginal cost of production: although these producers differ in terms of their international invoicing practices, they use the same production technology, they are subject to the same country-specific productivity shock tfp_t, and face the same prices for factors of production w_t and r_t.

Bundlers. Perfectly competitive bundlers combine inputs of domestic goods manufactured by producers and inputs of imported goods purchased from overseas to produce final consumption goods for both domestic markets, y_t^h, and export markets, y_t^x. Bundlers use Leontief technology

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[5]The total cost function of producer i is given by $tc(w_t, r_t, y_t^v)(i) \equiv r_t k_t^s(i) + w_t h_t(i) = [(1-\alpha)^{\sigma^y}w_t^{1-\sigma^y} + \alpha^{\sigma^y}r_t^{1-\sigma^y}]^{\frac{1}{1-\sigma^y}}\frac{y_t^v}{tfp_t}$.

to produce final goods:

$$y_t^h = min\{\frac{y_t^{hv}}{\kappa^{hv}}, \frac{mi_t^h}{1-\kappa^{hv}}\} \tag{45}$$

$$y_t^x = min\{\frac{y_t^{xv}}{\kappa^{xv}}, \frac{mi_t^x}{1-\kappa^{xv}}\}, \tag{46}$$

where κ^{hv} and κ^{xv} are parameters and the variables mi_t^h and mi_t^x denote the indices of imported inputs used to produce final consumption goods for domestic and export markets, respectively. Further, the indices of domestic inputs used in the production of consumption goods are given by:

$$y_t^{hv} \equiv [\int_0^1 (y_t^{hv}(i))^{\frac{\sigma^{hb}-1}{\sigma^{hb}}} di]^{\frac{\sigma^{hb}}{\sigma^{hb}-1}}$$

$$y_t^{xv} \equiv [\int_0^1 (y_t^{xv}(i))^{\frac{\sigma^{xb}-1}{\sigma^{xb}}} di]^{\frac{\sigma^{xb}}{\sigma^{xb}-1}},$$

where the parameter $\sigma^{hb} > 1$ is the elasticity of substitution among intermediate goods used in the production of domestic consumption brands and the parameter $\sigma^{xb} > 1$ is the elasticity of substitution among intermediate goods used in the production of export brands. Solving the cost minimisation problem yields the following:

$$\frac{\kappa^{hv}}{1-\kappa^{hv}} = \frac{y_t^{hv}}{mi_t^h} \tag{47}$$

$$\frac{\kappa^{xv}}{1-\kappa^{xv}} = \frac{y_t^{xv}}{mi_t^x} \tag{48}$$

$$mc_t^h = \kappa^{hv}phv_t + (1-\kappa^{hv})pm_t \tag{49}$$

$$mc_t^x = \kappa^{xv}\frac{pxv_t}{q_t} + (1-\kappa^{xv})pm_t, \tag{50}$$

where $phv_t \equiv [\int_0^1 (phv_t(i))^{1-\sigma^{hb}} di]^{\frac{1}{1-\sigma^{hb}}}$ and $pxv_t \equiv [\int_0^1 (pxv_t(i))^{1-\sigma^{xb}} di]^{\frac{1}{1-\sigma^{xb}}}$ are the associated price indices, where the latter is expressed in foreign currency.

2.2.1 Optimal price-setting

Producers. Producers can set one price for their products used in the production of domestic goods and another for those used in the production of export goods. Producers incur costs associated with adjusting nominal prices, and so prices are sticky in the short run. For tractability, we assume price adjustment costs in the spirit of Rotemberg (1982). That is firms will incur intangible price adjustment costs that do not affect their (economic) profits but enter their maximisation problem. Rotemberg-type price adjustments costs implies that in equilibrium producers will be identical in their price-setting behaviour. This is different to the Calvo price-setting since in a Calvo setting only a portion of firms would change prices at any given time. This then would lead to heterogeneity among firms in that, at any given time, firms would produce different levels of output, would have different demand schedules for factors of production, different levels of capital stock and rates of utilisation. Below, we introduce these costs in greater detail.

Total nominal profit at time t of a producer i from supplying to the domestic and export markets is given by:

$$\Pi_t^v(i) = PHV_t(i)y_t^{hv}(i) + \frac{PXVF_t(i)}{ER_t}y_t^{xv}(i) - TC_t(i), \tag{51}$$

where $TC_t(i) = W_t h_t(i) + R_t k_t^s(i)$ denotes the total nominal cost of production. We assume that firms maximise the discounted flow of $\widetilde{\Pi}_t^v(i)$, where

$$\widetilde{\Pi}_t^v(i) \equiv \Pi_t^v(i) - \Delta_t^{ph}(i) - \Delta_t^{px}(i), \tag{52}$$

where the variables $\Delta_t^{ph}(i)$ and $\Delta_t^{px}(i)$, defined below, denote the (intangible) costs associated with adjusting nominal domestic and export prices, respectively.

$$\Delta_t^{ph}(i) \equiv \frac{\chi^{hv}}{2}\Big[\frac{\frac{PHV_t(i)}{PHV_{t-1}(i)}}{(1+\dot{p}^{ss})^{1-\epsilon^{hv}}(\frac{PHV_{t-1}}{PHV_{t-2}})^{\epsilon^{hv}}} - 1\Big]^2 PHV_t y_t^{hv}. \tag{53}$$

Hence, the cost to the firm of adjusting prices is defined relative to the weighted average of steady-state rate of (gross) consumer price inflation and the last period's (gross) domestic producer price inflation. The parameter ϵ^{hv} is the weight attached to the latter. This structure implies faster price movements are more costly to the firm. We also assume that the cost of price adjustment increases in line with the aggregate revenue from supplying to the domestic market, $PHV_t y_t^{hv}$, which is taken as proxy for the size of the sector.[6] When $\chi^{hv} = 0$ prices are flexible. And when $\chi^{hv} > 0$, due to costs involved in adjustment, nominal prices will be sticky. Likewise, the variable $\Delta_t^{px}(i)$ is defined as the cost associated with adjusting nominal export prices expressed in domestic currency:

$$\Delta_t^{px}(i) \equiv \frac{\chi^{xv}}{2}\Big[\frac{(\frac{PXVF_t(i)}{PXVF_{t-1}(i)})}{(1+\dot{p}^{fss})^{1-\epsilon^{xv}}(\frac{PXVF_{t-1}}{PXVF_{t-2}})^{\epsilon^{xv}}} - 1\Big]^2 \frac{PXVF_t}{ER_t}y_t^{xv}, \tag{54}$$

where ϵ^{xv} is a parameter and $(1 + \dot{p}^{fss})$ denotes the steady-state (gross) consumer price inflation rate in the rest of the world.

The producer chooses its domestic and export price to maximise its expected total discounted profit flows:

$$E_t \sum_{r=0}^{\infty} \beta^r \lambda_{t+r}[PHV_{t+r}(i)y_{t+r}^{hv}(i) + \frac{PXVF_{t+r}(i)}{ER_{t+r}}y_{t+r}^{xv}(i) - TC_{t+r}(i) - \Delta_{t+r}^{ph}(i) - \Delta_{t+r}^{px}(i)],$$

subject to the production technology (equation (40)), price adjustment costs (equations (53) and (54)) and the total domestic and export demand for its product (equations (55) and (56)). The downward-sloping demand curve the producer faces in its domestic and export market is given by:

$$y_t^{hv}(i) = \Big(\frac{phv_t(i)}{phv_t}\Big)^{-\sigma^{hb}} y_t^{hv} \tag{55}$$

and

$$y_t^{xv}(i) = \Big(\frac{pxv_t(i)}{pxv_t}\Big)^{-\sigma^{xb}} y_t^{xv}, \tag{56}$$

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[6]In Ghironi (2002) the nominal price adjustment cost increases with firm revenue which is taken as a proxy for production size.

where $phv_t = \frac{PHV_t}{PC_t}$ and $pxv_t = PXV_t/PC_t$ as usual. And $PXV_t \equiv PXVF_t/ER_t$ is the export price expressed in domestic currency. The variable λ in the maximand is Home producer's discount factor (the marginal utility of wealth to the household). Finally, the firm takes the wage rate, the rental rate of capital, and the aggregate price indices as given.

The maximisation problem with respect to the domestic price, $phv_t(i)$, has the following first-order condition:

$$0 = \quad (1 - \sigma^{hb})(\frac{phv_t(i)}{phv_t})^{-\sigma^{hb}} y_t^{hv} - \frac{\partial TC_t(i)}{\partial PHV_t(i)} - \chi^{hv} \frac{phv_t}{phv_t(i)} y_t^{hv} \xi_t^{hv}(\xi_t^{hv} + 1)$$

$$+ \beta \chi^{hv} E_t \frac{\Lambda_{t+1}}{\Lambda_t} \frac{phv_{t+1}}{phv_t(i)} y_{t+1}^{hv} \xi_{t+1}^{hv}(\xi_{t+1}^{hv} + 1),$$

where $\Lambda_t \equiv PC_t \lambda_t$ as before. Using the demand function of the firm given above we can show that:

$$\frac{\partial TC_t(i)}{\partial PHV_t(i)} = \frac{\partial TC_t(i)}{\partial y_t^{hv}(i)} \frac{\partial y_t^{hv}(i)}{\partial PHV_t(i)}$$

$$= mc_t(-\sigma^{hb})(\frac{phv_t(i)}{phv_t})^{-\sigma^{hb}} \frac{y_t^{hv}}{phv_t(i)}.$$

In a symmetric equilibrium, the first-order condition yields the Home producer's optimal pricing equation as:

$$phv_t = \Psi_t^{hv} mc_t, \tag{57}$$

which equates the unit price charged by the producer to the product of the marginal cost (equation (44)) and a mark-up, Ψ_t^{hv}. As shown below, the mark-up depends on the firm's output as well as on today's pricing decision and on current and future cost of adjusting the output price. That is:

$$\Psi_t^{hv} = \sigma^{hb}\{(\sigma^{hb} - 1) + \chi^{hv}\Upsilon_t^{hv}\}^{-1}, \tag{58}$$

and where

$$\Upsilon_t^{hv} \equiv \xi_t^{hv}(\xi_t^{hv} + 1) - \beta E_t\{\frac{\Lambda_{t+1}}{\Lambda_t} \frac{phv_{t+1}}{phv_t} \frac{y_{t+1}^{hv}}{y_{t+1}^{hv}} \xi_{t+1}^{hv}(\xi_{t+1}^{hv} + 1)\}$$

$$\cong \xi_t^{hv}(\xi_t^{hv} + 1) - \frac{1}{rrg_t} E_t\{\frac{phv_{t+1}}{phv_t} \frac{y_{t+1}^{hv}}{y_{t+1}^{hv}} \xi_{t+1}^{hv}(\xi_{t+1}^{hv} + 1)\}$$

$$\xi_t^{hv} \equiv \frac{1 + \dot{p}_t^{hv}}{(1 + \dot{p}^{ss})^{1-\epsilon^{hv}}(1 + \dot{p}_{t-1}^{hv})^{\epsilon^{hv}}} - 1.$$

Consistent with our notation $1 + \dot{p}_t^{hv} \equiv \frac{PHV_t}{PHV_{t-1}} = (1 + \dot{p}_t)\frac{phv_t}{phv_{t-1}}$. Υ_t^{hv} reflects the firm's incentive to smooth prices over time. If $\chi^{hv} = 0$, that is if prices are fully flexible, then $\Psi_t^{hv} = \sigma^{hb}/(\sigma^{hb} - 1)$ collapses to the familiar constant elasticity mark-up. If $\chi^{hv} \neq 0$, then prices are sticky and this will give rise to endogenous fluctuations of the mark-up.

The maximisation problem with respect to the export price, $pxv_t(i)$, has the following first-order

condition:

$$0 = \quad (1 - \sigma^{xb})\left(\frac{pxv_t(i)}{pxv_t}\right)^{-\sigma^{xb}} - \frac{\partial TC_t(i)}{\partial PXVF_t(i)}\frac{ER_t}{y_t^{xv}} - \chi^{xv}\frac{pxv_t}{pxv_t(i)}\xi_t^{xv}(\xi_t^{xv} + 1)$$

$$+ \beta\chi^{xv}E_t\frac{\Lambda_{t+1}}{\Lambda_t}\frac{pxv_{t+1}}{pxv_t(i)}\frac{y_{t+1}^{xv}}{y_t^{xv}}\xi_{t+1}^{xv}(\xi_{t+1}^{xv} + 1).$$

As before, using the export demand function given above, we can show that:

$$\frac{\partial TC_t(i)}{\partial PXVF_t(i)} = \frac{\partial TC_t(i)}{\partial y_t^{xv}(i)}\frac{\partial y_t^{xv}(i)}{\partial PXVF_t(i)}$$

$$= mc_t(-\sigma^{xb})\left(\frac{pxv_t(i)}{pxv_t}\right)^{-\sigma^{xb}}\frac{y_t^{xv}}{ER_t pxv_t(i)}.$$

In a symmetric equilibrium, the first-order condition yields the optimal export pricing equation as:

$$pxv_t = \Psi_t^{xv}mc_t. \tag{59}$$

The equation for the mark-up, Ψ_t^{xv}, is given below:

$$\Psi_t^{xv} = \sigma^{xb}\{(\sigma^{xb} - 1) + \chi^{xv}\Upsilon_t^{xv}\}^{-1},$$

and where

$$\Upsilon_t^{xv} \equiv \xi_t^{xv}(\xi_t^{xv} + 1) - \beta E_t\{\frac{\Lambda_{t+1}}{\Lambda_t}\frac{pxv_{t+1}}{pxv_t}\frac{y_{t+1}^{xv}}{y_t^{xv}}\xi_{t+1}^{xv}(\xi_{t+1}^{xv} + 1)\}$$

$$\cong \xi_t^{xv}(\xi_t^{xv} + 1) - \frac{1}{rrg_t}\{\frac{pxv_{t+1}}{pxv_t}\frac{y_{t+1}^{xv}}{y_t^{xv}}\xi_{t+1}^{xv}(\xi_{t+1}^{xv} + 1)\}$$

$$\xi_t^{xv} \equiv \frac{1 + \dot{p}_t^{xvf}}{(1 + \dot{p}^{fss})^{1-\epsilon^{xv}}(1 + \dot{p}_{t-1}^{xvf})^{\epsilon^{xv}}} - 1.$$

where $1 + \dot{p}_t^{xvf} \equiv \frac{PXVF_t}{PXVF_{t-1}} = (1 + \dot{p}_t^f)\frac{pxvf_t}{pxvf_{t-1}}$. Here, $1 + \dot{p}_t^f \equiv \frac{PCF_t}{PCF_{t-1}}$ denotes the (gross) consumer price inflation in the rest of the world in period t.

Bundlers. Bundlers are perfectly competitive. Thus, the (relative) price that bundlers charge for their output sold in the domestic market, ph_t, and in the export market, px_t, would be equal to their respective (real) marginal cost of production shown in **(49)** and **(50)**. That is

$$ph_t = \kappa^{hv}phv_t + (1 - \kappa^{hv})pm_t \tag{60}$$

$$px_t = \kappa^{xv}pxv_t + (1 - \kappa^{xv})pm_t. \tag{61}$$

Also, notice that the zero-profit condition of bundlers would imply that:

$$ph_t y_t^h = phv_t y_t^{hv} + pm_t mi_t^h \tag{62}$$

$$px_t y_t^x = pxv_t y_t^{xv} + pm_t mi_t^x. \tag{63}$$

Finally, the total demand faced by bundlers supplying to the domestic market is given by:

$$\int_0^1 ch_t(j)dj + \int_0^1 I_t(j)dj + g_t = ch_t + I_t + g_t.$$

The equality follows from symmetric equilibrium. We assume that the total export demand faced by bundlers is given by:

$$x_t = \kappa^x \big(\frac{q_t px_t}{pxf_t}\big)^{-\sigma^x} cf_t, \tag{64}$$

where the variable cf_t is the aggregate (exogenous) export demand in the rest of the world and the variable pxf_t denotes the (exogenous) index of world export prices (relative to PCF_t). The elasticity of demand for Home exports is given the parameter $\sigma^x > 1$.

2.3 Government budget constraint

In our set-up, Ricardian equivalence holds. Hence, without loss of generality, we can assume that the government runs a balanced budget every period and that the issuance of domestic government bond is zero in each period.[7] Then, in aggregate, total lump-sum tax/transfer payments are equal to total government spending less the seignorage revenue of the government:

$$PC_t \tau_t = PH_t g_t - (MON_{A,t} - MON_{A,t-1}).$$

Hence, we assume that the government adjusts taxes/transfers to ensure that its budget is balanced every period.

2.4 Monetary policy rule

We assume that the central bank conducts monetary policy through changes in the interest rate. The monetary policy reaction function is a 'Taylor rule' with smoothing:

$$\frac{1 + rg_t}{1 + rg^{ss}} = \big(\frac{1 + rg_{t-1}}{1 + rg^{ss}}\big)^{\theta^{rg}} \{[\frac{(1 + \dot{p}_t)}{(1 + \dot{p}^{ss})}]^{\theta^p} \big(\frac{y_t^v}{y^{v,ss} tfp_t}\big)^{\theta^y}\}^{1-\theta^{rg}}, \tag{65}$$

where rg^{ss} is the steady-state level for the short-term nominal interest rate, $y^{v,ss}$ is the steady-state level of output, and $y^{v,ss} tfp_t$ is a simple measure of potential output. θ^{rg}, θ^p, and θ^y are parameters. We also assume that the central bank supplies whatever amount of money is necessary to ensure that the above equation holds.

2.5 Market clearing conditions

We impose equilibrium market clearing conditions for the goods, money, labour, and capital markets under the assumption that this equilibrium is symmetric (and we will retain this assumption hereafter).

Goods market. In a symmetric equilibrium all producers charge the same unit price (hence supply the same quantity) for their output consumed domestically as well as that consumed

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[7]This is because in the presence of state-contingent claims the bond market is redundant. For a detailed discussion see Chapter 5 in Obstfeld and Rogoff (1996).

abroad. Likewise, bundlers charge the same unit price (hence supply the same quantity) for their output sold in domestic markets as well as that sold in export markets. First, notice that the output of producers must be purchased by bundlers:

$$y_t^v = y_t^{hv} + y_t^{xv}, \tag{66}$$

where quantities supplied to domestic (y^{hv}) and export markets (y^{xv}) satisfy the following:

$$y_t^{hv} = y_t^h - mi_t^h$$
$$y_t^{xv} = y_t^x - mi_t^x.$$

Then we have:

$$y_t^v = y_t^h + y_t^x - (mi_t^h + mi_t^x) \tag{67}$$

The supply of final domestic goods (y_t^h) must satisfy domestic demand and likewise the supply of final export goods (y_t^x) must satisfy export demand:

$$y_t^h = ch_t + I_t + g_t \tag{68}$$
$$y_t^x = x_t, \tag{69}$$

where investment I_t (measured inclusive of adjustment costs) is given by household's capital accumulation equation **(16)**. Also notice that output of producers can be expressed as:

$$y_t^{hv} = \kappa^{hv} y_t^h$$
$$y_t^{xv} = \kappa^{xv} y_t^x.$$

Putting this together implies that the market clearing condition becomes

$$y_t^v = \kappa^{hv}(ch_t + I_t + g_t) + \kappa^{xv} x_t \tag{70}$$
$$= \kappa^{hv} y_t^h + \kappa^{xv} y_t^x. \tag{71}$$

Labour market. The capital market clearing condition requires that total labour services hired by all firms equals to the total labour services supplied by all households at the wage rate set by households.

Capital market. The capital market clearing condition requires that total capital services rented by all firms equals to the total capital services supplied by all households.

Money market. The money market clearing condition simply requires that the supply of real money balances should equal to the demand for real money balances. The nominal interest rate is determined by the monetary policy rule and thus the money supply adjusts endogenously to meet the money demand at the prevailing interest rate.

Financial and asset markets. At an equilibrium financial assets must be in zero net supply.

2.6 Some international variables

Consumption-based real exchange rate. The (consumption-based) real exchange rate is defined previously as units of domestic consumption per units of foreign consumption. That is:

$$q_t = \frac{PC_t ER_t}{PCF_t}.$$

A decrease in q_t, a real depreciation, implies that goods are becoming relatively more expensive in the rest of the world.

The terms of trade. The terms of trade is defined as the price of the imported good relative to the price of exported good, both in terms of Home currency:

$$ToT_t = \frac{pm_t}{px_t}.$$

An increase in ToT_t, a worsening in the domestic economy's terms of trade, implies that imports have become relatively more expensive than exports.

Import prices. It is assumed that import prices are set in domestic currency and in staggered contracts as in Calvo (1983). That is, a fraction of import prices (ψ^{pm}) are reset each period. The remaining fraction continues to charge the old price. Prices set by the remaining fraction of firms are increased in line with a weighted average of the steady-state inflation rate and the lagged import price inflation rate, where the parameter ϵ^m denotes the weight attached to the latter:

$$\xi_{t,t+r}^m = \begin{cases} 1 & \text{if } r = 0 \\ (1 + \dot{p}^{ss})^{1-\epsilon^m} (\frac{PM_{t+r-1}}{PM_{t+r-2}})^{\epsilon^m} \xi_{t,t+r-1}^m & \text{if } r \geq 1. \end{cases}$$

In any period t which the firm i is able to reset its price, $PM_t(i)$, it maximises the expected discounted nominal profit flows assuming that this newly set price remains fixed along the path:

$$E_t \sum_{r=0}^{\infty} \beta^r (1 - \psi^{pm})^r [\xi_{t,t+r}^m PM_t(i) cm_{t+r}(i) - TC_{t+r}^f(i)],$$

$cm_{t+r}(i)$ and $TC_{t+r}^f(i)$ denote total output demanded and nominal total costs firm i faces at time $t + r$, respectively.[8] And the probability that the firm's price remains fixed for the next r periods is $(1 - \psi^{pm})^r$. Finally, note that the level of demand the monopolistically competitive firm i faces at date $t + r$ is given by:

$$cm_{t+r}(i) = (\frac{\xi_{t,t+r}^m PM_t(i)}{PM_{t+r}})^{-\sigma^{mb}} cm_{t+r},$$

where cm_{t+r} is the aggregate demand in the economy at time $t + r$.

---

[8]Notice that the foreign firm's discount factor does not appear in this equation. This is because, we do not explicitly model foreign agents and their discount factor does not affect the dynamics of the linearised pricing equations.

The maximisation problem has the following first-order condition:

$$0 = E_t \sum_{r=0}^{\infty} \beta^r (1 - \psi^{pm})^r [(1 - \sigma^{mb})(\frac{\xi_{t,t+r}^m \widetilde{PM}_t(i)}{PM_{t+r}})^{-\sigma^{mb}} cm_{t+r} - \frac{\partial TC_{t+r}^f(i)}{\partial PM_t(i)}].$$

Using the demand function of the firm given above we can show that:

$$\frac{\partial TC_{t+r}^f(i)}{\partial PM_t(i)} = \frac{\partial TC_{t+r}^f(i)}{\partial cm_{t+r}(i)} \frac{\partial cm_{t+r}(i)}{\partial PM_t(i)}$$

$$= [\frac{\partial TC_{t+r}^f(i)}{\partial cm_{t+r}(i)} \frac{1}{PM_{t+r}}](-\sigma^{mb})(\frac{\xi_{t,t+r}^m \widetilde{PM}_t(i)}{PM_{t+r}})^{-(\sigma^{mb}+1)} cm_{t+r}. \tag{72}$$

Let $rmc_{t+r}^f(i)$ denote the real marginal cost of the foreign firm at time $t + r$, the first term in squared brackets on the right-hand side of equation **(72)**. Then, the first-order condition becomes:

$$0 = E_t \sum_{r=0}^{\infty} \beta^r (1 - \psi^{pm})^r [(1 - \sigma^{mb})(\frac{\xi_{t,t+r}^m \widetilde{PM}_t(i)}{PM_{t+r}})^{-\sigma^{mb}} cm_{t+r}]$$

$$+ E_t \sum_{r=0}^{\infty} \beta^r (1 - \psi^{pm})^r [\sigma^{mb} rmc_{t+r}^f(i)(\frac{\xi_{t,t+r}^m \widetilde{PM}_t(i)}{PM_{t+r}})^{-(\sigma^{mb}+1)} cm_{t+r}].$$

We can rewrite this expression in terms of the firm's price relative to the average price index as:

$$0 = E_t \sum_{r=0}^{\infty} \beta^r (1 - \psi^{pm})^r [(1 - \sigma^{mb})(\frac{\xi_{t,t+r}^m PM_t}{PM_{t+r}})^{-\sigma^{mb}} (\frac{\widetilde{PM}_t(i)}{PM_t})^{-\sigma^{mb}} cm_{t+r}]$$

$$+ E_t \sum_{r=0}^{\infty} \beta^r (1 - \psi^{pm})^r [\sigma^{mb} rmc_{t+r}^f(i)(\frac{\xi_{t,t+r}^m PM_t}{PM_{t+r}})^{-(\sigma^{mb}+1)} (\frac{\widetilde{PM}_t(i)}{PM_t})^{-(\sigma^{mb}+1)} cm_{t+r}].$$

Note that all firms that set prices at this date are identical. Thus, they will all charge the same price and produce at the same level of output. If we further assume that firms purchase inputs in competitive markets and employ constant returns to scale technology then the (real) marginal cost is equal to the average cost and is identical across all firms. The optimal price of any firm setting prices at time t is then given by:

$$\widetilde{PM}_t = PM_t \frac{\sigma^{mb}}{\sigma^{mb} - 1} \frac{E_t \sum_{r=0}^{\infty} \beta^r (1 - \psi^{pm})^r [rmc_{t+r}^f (\frac{\xi_{t,t+r}^m PM_t}{PM_{t+r}})^{-(\sigma^{mb}+1)} cm_{t+r}]}{E_t \sum_{r=0}^{\infty} \beta^r (1 - \psi^{pm})^r [(\frac{\xi_{t,t+r}^m PM_t}{PM_{t+r}})^{-\sigma^{mb}} cm_{t+r}]}.$$

Then, we can rewrite this equation in a simpler form as:

$$\widetilde{PM}_t = PM_t \frac{\sigma^{mb}}{\sigma^{mb} - 1} \frac{\Xi_t^m}{\Theta_t^m}, \tag{73}$$

where

$$\Xi_t^m = rmc_t^f cm_t + \beta(1 - \psi^{pm}) E_t (\frac{\xi_{t,t+1}^m}{1 + \dot{p}_{t+1}^m})^{-(\sigma^{mb}+1)} \Xi_{t+1}^m$$

$$\Theta_t^m = cm_t + \beta(1 - \psi^{pm}) E_t (\frac{\xi_{t,t+1}^m}{1 + \dot{p}_{t+1}^m})^{-\sigma^{mb}} E_t \Theta_{t+1}^m,$$

and where

$$\xi_{t,t+1}^m = (1 + \dot{p}^{ss})^{1-\epsilon^m} (1 + \dot{p}_t^m)^{\epsilon^m}.$$

Note that consistent with our notation $1 + \dot{p}_t^m \equiv \frac{PM_t}{PM_{t-1}} = (1 + \dot{p}_t)\frac{pm_t}{pm_{t-1}}$. The aggregate import price index at time t can now be written as:

$$
\begin{aligned}
PM_t^{1-\sigma^{mb}} &\equiv \int_0^1 (PM_t(i))^{1-\sigma^{mb}} di \\
&= (1 - \psi^{pm}) \int_0^1 (\xi_{t-1,t}^m PM_{t-1}(i))^{1-\sigma^{mb}} di + \psi^{pm}(\widetilde{PM}_t)^{1-\sigma^{mb}} \\
&= (1 - \psi^{pm})(\xi_{t-1,t}^m PM_{t-1})^{1-\sigma^{mb}} + \psi^{pm}(\widetilde{PM}_t)^{1-\sigma^{mb}}.
\end{aligned}
\tag{74}
$$

where

$$
\xi_{t-1,t}^m = (1 + \dot{p}^{ss})^{1-\epsilon^m}(1 + \dot{p}_{t-1}^m)^{\epsilon^m}.
$$

Dividing both sides of the equation by PC_t we obtain the relative import price index, first defined in equation **(9)**:

$$
pm_t^{1-\sigma^{mb}} = (1 - \psi^{pm})(\xi_{t-1,t}\frac{pm_{t-1}}{1+\dot{p}_t})^{1-\sigma^{mb}} + \psi^{pm}(\widetilde{pm}_t)^{1-\sigma^{mb}}.
$$

Hence, the average import price at date t is a weighted average of the newly set prices and the lagged price level. Finally we proxy the cost of imports by the term:

$$
rmc_t^f = \frac{pxf_t}{q_t pm_t},
$$

which reflects the assumption that the exporting firms in the rest of the world purchase goods on world markets at price pxf_t.

2.7 Net foreign assets

The household's budget constraint evaluated at a symmetric equilibrium can be found by substituting the government budget constraint into the household budget constraint:

$$
PC_t c_t + PH_t I_t + \frac{BF_t}{ER_t} - (1 + rf_{t-1})\frac{BF_{t-1}}{ER_t} = W_t h_t + R_t k_t^s + \Pi_t^v - PH_t g_t.
$$

Noticing that

$$
\Pi_t^v = PHV_t y_t^{hv} + \frac{PXV_t}{ER_t} y_t^{xv} - W_t h_t - R_t k_t^s,
$$

and substituting the above expression into the budget constraint we obtain:

$$
PC_t c_t + PH_t I_t + \frac{BF_t}{ER_t} - (1 + rf_{t-1})\frac{BF_{t-1}}{ER_t} = PHV_t y_t^{hv} + \frac{PXV_t}{ER_t} y_t^{xv} - PH_t g_t.
$$

Using the zero-profit condition of bundlers:

$$
\begin{aligned}
PHV_t y_t^{hv} &= PH_t y_t^h - PM_t mi_t^h \\
\frac{PXV_t}{ER_t} y_t^{xv} &= PX_t y_t^x - PM_t mi_t^x,
\end{aligned}
$$

together with the goods market clearing condition,

$$
y_t^h = ch_t + I_t + g_t,
$$

we can show that:

$$PC_t c_t + \frac{BF_t}{ER_t} - (1 + rf_{t-1})\frac{BF_{t-1}}{ER_t} = PH_t ch_t + PX_t y_t^x - PM_t(mi_t^h + mi_t^x).$$

Then, using the expression for total consumption expenditure, $PC_t c_t = PH_t ch_t + PM_t cm_t$, yields the equation for net foreign asset accumulation as a function of interest income, export supply, and import demand:

$$\frac{BF_t}{ER_t} = (1 + rf_{t-1})\frac{BF_{t-1}}{ER_t} + PX_t y_t^x - PM_t(cm_t + mi_t^h + mi_t^x).$$

In real terms this equation can be rewritten as:

$$\frac{bf_t}{q_t} = \frac{bf_{t-1}}{(1 + \dot{p}_t^f)q_t} + rf_{t-1}\frac{bf_{t-1}}{(1 + \dot{p}_t^f)q_t} + px_t y_t^x - pm_t(cm_t + mi_t^h + mi_t^x)$$

$$nfa_t = nfa_{t-1}\frac{q_{t-1}}{(1 + \dot{p}_t^f)q_t} + rf_{t-1}nfa_{t-1}\frac{q_{t-1}}{(1 + \dot{p}_t^f)q_t} + px_t y_t^x$$

$$- pm_t(cm_t + mi_t^h + mi_t^x). \tag{75}$$

where $nfa_t \equiv \frac{bf_t}{q_t}$ denotes household's net foreign asset holdings at time t. The domestic capital account by definition equals the change in the net foreign asset position. That is:

$$ca_t \equiv \frac{bf_{t-1}}{(1 + \dot{p}_t^f)q_t} - \frac{bf_t}{q_t} \equiv nfa_{t-1}\frac{q_{t-1}}{(1 + \dot{p}_t^f)q_t} - nfa_t,$$

Expression **(75)** shows that the capital account equals in magnitude and opposite in sign to the current account, where the current account is defined as the sum of foreign asset returns (the second term on the right-hand side) and net trade (the third term on the right-hand side).

2.8 Model variables

All in all, in the model, we have 38 variables determined during time t: phv_t, pxv_t, $pxvf_t$, ph_t, px_t, pm_t, Ψ_t^{hv}, Ψ_t^{xv}, $U_{c,t}$, c_t, ch_t, cm_t, I_t, y_t^v, y_t^{hv}, y_t^{xv}, y_t^h, y_t^x, x_t, mi_t^h, mi_t^x, w_t, r_t, h_t, k_t^s, k_t, z_t, mc_t, rg_t, rrg_t, mon_t, q_t, bf_t, $\dot{p}_t$, $\dot{p}_t^{hv}$, $\dot{p}_t^{xvf}$, $\dot{p}_t^m$, $\dot{p}_t^w$. There are 18 state variables that are predetermined as of time t: c_{t-1}, bf_{t-1}, q_{t-1}, k_{t-1}, k_{t-2}, w_{t-1}, phv_{t-1}, $pxvf_{t-1}$, pm_{t-1}, rg_{t-1}, $\dot{p}_{t-1}$, $\dot{p}_{t-2}$, $\dot{p}_{t-3}$, $\dot{w}_{t-1}$, $\dot{p}_{t-1}^{hv}$, $\dot{p}_{t-1}^{xvf}$, $\dot{p}_{t-1}^m$, rf_{t-1}. We treat the share of government expenditure in domestic output as exogenous. Since domestic economy is a small open economy, the rest of the world variables are also treated as exogenous: pxf_t, pcf_t, $\dot{p}_t^f$, $\dot{p}_t^{fss}$, rf_t, rrf_t, cf_t. These 38 endogenous variables are determined by a system of 38 equations shown in Table A.

3 Steady state

In this section, we solve for the steady-state level of the model's variables as functions of the model's parameters. We summarise the model's steady-state equations in Table B. The steady-state level of nominal interest rate is determined by the consumption Euler equation **(33)** evaluated at the steady state:

$$rg^{ss} = \frac{1 - \beta}{\beta}.$$

Without loss of generality, we can set:

$$pm^{ss} = ph^{ss}.$$

This implies:

$$pxf^{ss}\frac{\sigma^{mb}}{\sigma^{mb} - 1} = ph^{ss}q^{ss}.$$

Then, the expression for consumer price index reduces to:

$$1 = \frac{1}{\kappa^c}[(1 - \psi^m)^{\sigma^m} + (\psi^m)^{\sigma^m}]^{\frac{1}{1-\sigma^m}}ph^{ss}.$$

If we set $\kappa^c = [(1 - \psi^m)^{\sigma^m} + (\psi^m)^{\sigma^m}]^{\frac{1}{1-\sigma^m}}$, then we have:

$$ph^{ss} = pm^{ss} = 1.$$

Further, the money market clearing means that equation **(36)** can then be restated as:

$$mon^{ss} = \kappa^{mon}[U_{c,ss}(1 - \beta)]^{-\sigma^c}.$$

Given the steady-state money holdings, we can solve for the steady-state consumption:

$$c^{ss} = [\frac{mon^{ss}(1 - \beta)^{-\sigma^c}}{\kappa^{mon}}]^{(\psi^{hab} - \psi^{hab}\sigma^c - 1)}, \tag{76}$$

where κ^{mon} is a parameter. Using the steady-state versions of equations **(12)** and **(13)** and the expression for κ^c given above we can then solve for steady-state domestic and import consumption:

$$ch^{ss} = \frac{(1 - \psi^m)^{\sigma^m}c^{ss}}{(1 - \psi^m)^{\sigma^m} + (\psi^m)^{\sigma^m}} \tag{77}$$

$$cm^{ss} = \frac{(\psi^m)^{\sigma^m}c^{ss}}{(1 - \psi^m)^{\sigma^m} + (\psi^m)^{\sigma^m}}. \tag{78}$$

Since $ph^{ss} = pm^{ss} = 1$, the pricing equation for bundlers that supply to the domestic market becomes:

$$1 = \kappa^{hv}phv^{ss} + (1 - \kappa^{hv}),$$

which then implies that:

$$phv^{ss} = 1.$$

At the steady state, the optimal price charged by producers supplying to the domestic market reduces to a constant mark-up over marginal cost, where this mark-up is given by:

$$\Psi^{hv,ss} = \frac{\sigma^{hb}}{\sigma^{hb} - 1}.$$

Then, evaluating equation **(57)** at the steady state we can solve for the steady-state marginal cost of producers:

$$mc^{v,ss} = \frac{\sigma^{hb} - 1}{\sigma^{hb}}.$$

Likewise, at the steady state, the optimal price of producers supplying to the export market is a constant mark-up over marginal cost, where this mark-up is given by:

$$\Psi^{xv,ss} = \frac{\sigma^{xb}}{\sigma^{xb} - 1}.$$

Then, equation (59) implies that:

$$pxv^{ss} = (\frac{\sigma^{xb}}{\sigma^{xb-1}})(\frac{\sigma^{hb} - 1}{\sigma^{hb}}).$$

Then, we can solve for the steady-state export price using equation (61):

$$px^{ss} = \kappa^{xv}(pxv^{ss} - 1) + 1.$$

If we further set $\sigma^{hb} = \sigma^{xb}$, we have

$$pxv^{ss} = px^{ss} = 1.$$

Now we turn to the export demand equation equation (56). Without loss of generality, we can set the steady-state consumption in the rest of the world to one, ie $cf^{ss} = 1$. And $pm^{ss} = 1$ implies that world export prices should satisfy:

$$pm^{ss} = \frac{\sigma^{mb}}{\sigma^{mb} - 1} pxf^{ss}.$$

Then, the export demand equation becomes:

$$x^{ss} = \kappa^x [\frac{q^{ss}}{pxf^{ss}}]^{-\sigma^x}.$$

Further, if we set $\kappa^x = x^{ss}(pxf^{ss})^{-\sigma^x}$, we have:

$$q^{ss} = 1.$$

Next we turn to the equation for net foreign asset accumulation at Home equation (75). Evaluated at the steady state, this equation yields:

$$nfa^{ss} = \frac{\beta}{1 - \beta}[cm^{ss} + mi^{h,ss} + mi^{x,ss} - y^{x,ss}].$$

Notice that $pm^{ss} = px^{ss} = q^{ss} = 1$. Hence, given nfa^{ss} we can pin down the current account position at the steady state. We can rewrite the above equation in terms of domestic demand and supply. To do this we make use of market clearing conditions in the goods market. First, we note that the output of producers must be purchased by bundlers equation (67):

$$
\begin{aligned}
y^{v,ss} &= y^{hv,ss} + y^{xv,ss} & \text{(79)} \\
&= y^{h,ss} + y^{x,ss} - (mi^{h,ss} + mi^{x,ss}) & \text{(80)} \\
&= \kappa^{hv} y_t^h + \kappa^{xv} y_t^x. & \text{(81)}
\end{aligned}
$$

Likewise, the supply of domestic goods must satisfy the domestic demand equation (68):

$$y^{h,ss} = ch^{ss} + I^{ss} + g^{ss}. \tag{82}$$

Finally, the supply of final export goods must satisfy the export demand equation **(69)**:

$$y^{x,ss} = x^{ss}.$$

Using these clearing conditions and taking the steady-state government spending to domestic output ratio as given, $\overline{g} = g^{ss}/y^{h,ss}$, we obtain:

$$nfa^{ss} = (\frac{\beta}{1-\beta})[cm^{ss} + \frac{ch^{ss} + I^{ss}}{1-\overline{g}} - y^{v,ss}].$$

The right-hand side of the above expression is the net total demand for domestic goods. Re-arranging we obtain:

$$y^{v,ss} = cm^{ss} + \frac{ch^{ss} + I^{ss}}{1-\overline{g}} - \frac{1-\beta}{\beta}nfa^{ss}.$$

Without loss of generality, we can set:

$$y^{v,ss} = 1.$$

Given the steady-state net foreign asset holdings, we can then solve for the steady-state investment:

$$I^{ss} = (1-\overline{g})[1 - cm^{ss} - \frac{ch^{ss}}{1-\overline{g}} + \frac{1-\beta}{\beta}nfa^{ss}].$$

Given I^{ss} equation **(82)** provides us with $y^{h,ss}$. We can then solve for:

$$y^{hv,ss} = \kappa^{hv}y^{h,ss}$$
$$mi^{h,ss} = (1-\kappa^{hv})y^{h,ss}.$$

Moreover, from equation **(79)** we know that:

$$y^{xv,ss} = y^{v,ss} - y^{hv,ss},$$

which then can be used to solve for:

$$y^{x,ss} = \frac{y^{xv,ss}}{\kappa^{xv}}$$
$$mi^{x,ss} = (1-\kappa^{xv})y^{x,ss}.$$

Now, we turn to the factor market. Evaluated at the steady state equation **(39)** implies that the rental rate at the steady state is:

$$r^{ss} = \chi^z. \tag{83}$$

Further, equation **(38)** tells us that capital at the steady state depreciates at a rate equal to $\chi^z + 1 - \frac{1}{\beta}$. This then implies:

$$\chi^z = \delta - 1 + \frac{1}{\beta}. \tag{84}$$

Additionally, equation **(16)** evaluated at the steady state implies:

$$k^{ss} = \frac{I^{ss}}{\delta}. \tag{85}$$

That is households should keep investing at the rate capital depreciates in order to keep the stock of capital at its steady-state level. Note that at the steady state the rate of capital utilisation is set

to one, that is:

$$z^{ss} = 1.$$

Hence, the steady-state capital services supplied by households equals the available capital stock,

$$k^{s,ss} = k^{ss}.$$

Re-arranging the expressions for conditional factor demands equations **(42)** and **(43)** and the expression for the marginal cost of production equation **(44)**, we obtain:

$$\frac{w^{ss}}{mc^{ss}} = (tfp^{ss})^{\frac{\sigma^y-1}{\sigma^y}}(1-\alpha)(\frac{h^{ss}}{y^{v,ss}})^{\frac{-1}{\sigma^y}} \tag{86}$$

$$\frac{r^{ss}}{mc^{ss}} = (tfp^{ss})^{\frac{\sigma^y-1}{\sigma^y}}\alpha(\frac{k^{ss}}{y^{v,ss}})^{\frac{-1}{\sigma^y}}. \tag{87}$$

Using equation **(87)**, we can solve the steady-state total factor productivity:

$$tfp^{ss} = (\frac{r^{ss}}{\alpha mc^{ss}})^{\frac{\sigma^y}{\sigma^y-1}}(k^{ss})^{\frac{1}{\sigma^y-1}}.$$

Note that $y^{v,ss} = 1$. Given k^{ss} and $y^{v,ss}$, shown above, we can use equation **(40)** evaluated at the steady state to solve for the steady-state hours worked:

$$h^{ss} = [\frac{1}{1-\alpha}(\frac{y^{v,ss}}{tfp^{ss}})^{\frac{\sigma^y-1}{\sigma^y}} - \frac{\alpha}{1-\alpha}(k^{ss})^{\frac{\sigma^y-1}{\sigma^y}}]^{\frac{\sigma^y}{\sigma^y-1}}. \tag{88}$$

Then, equations **(86)** and **(87)** can be used to solve for steady-sate wages:

$$w^{ss} = \frac{1-\alpha}{\alpha}r^{ss}(\frac{k^{ss}}{h^{ss}})^{\frac{1}{\sigma^y}} \tag{89}$$

Finally, notice that at the steady-state aggregate wage index equals to the optimal wage rate set by individuals, ie $w^{ss} = \widetilde{w}^{ss}$. Then, given w^{ss} and h^{ss}, equation **(25)** evaluated at the steady state provides us with an expression for κ^h that must hold in equilibrium:

$$\kappa^h = (\frac{\sigma^w}{\sigma^w-1})^{-\sigma^h}(h^{ss})^{-1}(w^{ss})^{\sigma^h}(U_{c,ss})^{\sigma^h}.$$

4 The linearised model

We use standard log-linearisation techniques in order to analyse the dynamics of the model. Namely, we take a first-order Taylor approximation of the model's equations around the non-stochastic steady state. The hat above a variable denotes its percentage deviation from steady state. The ss in subscript denotes the steady-state level of a variable. The linearised model equations are shown in Table C and discussed in further detail below.

The consumption equation is given by:

$$\hat{c}_t = \frac{1}{1+\psi^{hab}-\psi^{hab}\sigma^c}E_t\hat{c}_{t+1} + \frac{\psi^{hab}-\psi^{hab}\sigma^c}{1+\psi^{hab}-\psi^{hab}\sigma^c}\hat{c}_{t-1} - \frac{\sigma^c}{1+\psi^{hab}-\psi^{hab}\sigma^c}r\hat{r}g_t. \tag{90}$$

The equation that governs the accumulation of capital stock, with adjustment costs, is given by:

$$\hat{k}_t - \hat{k}_{t-1} = \frac{\beta}{1+\beta\epsilon^k}(E_t\hat{k}_{t+1} - \hat{k}_t) + \frac{\epsilon^k}{1+\beta\epsilon^k}(\hat{k}_{t-1} - \hat{k}_{t-2}) + \frac{\beta\chi^z}{\chi^k(1+\beta\epsilon^k)}E_t\hat{r}_{t+1}$$

$$+ \frac{\beta(1-\delta)}{\chi^k(1+\beta\epsilon^k)}E_t\hat{ph}_{t+1} - \frac{1}{\chi^k(1+\beta\epsilon^k)}\hat{ph}_t - \frac{1}{\chi^k(1+\beta\epsilon^k)}r\hat{r}g_t \quad \textbf{(91)}$$

The investment equation is given by:

$$\hat{I}_t = \frac{1}{\delta}(\hat{k}_t - \hat{k}_{t-1}) + \hat{k}_{t-1} + \frac{\chi^z}{1-\delta}\hat{z}_t. \quad \textbf{(92)}$$

The optimal utilisation rate of capital is given by:

$$\hat{z}_t = \frac{1}{\sigma^z}(\hat{r}_t - \hat{ph}_t). \quad \textbf{(93)}$$

The utilisation rate increases with the rental rate of capital services as it becomes more profitable to use the existing capital stock more intensively but decreases with the price of investment goods as it becomes more costly to replace depleted capital.

The domestic producer price inflation equation in the intermediate goods sector (also known as the New Keynesian Phillips Curve) is given by:

$$\dot{p}_t^{hv} - \dot{p}^{ss} = \frac{\sigma^{hb}-1}{\chi^{hv}(1+\beta\epsilon^{hv})}r\hat{m}c_t^{hv} + \frac{\beta}{1+\beta\epsilon^{hv}}(E_t\dot{p}_{t+1}^{hv} - \dot{p}_t^{ss}) + \frac{\epsilon^{hv}}{1+\beta\epsilon^{hv}}(\dot{p}_{t-1}^{hv} - \dot{p}_t^{ss}), \quad \textbf{(94)}$$

where $r\hat{m}c_t^{hv} = \hat{m}c_t - \hat{phv}_t$ and $\dot{p}_t^{hv} - \dot{p}^{ss}$ are the percentage deviations of real marginal cost and (gross) domestic intermediate goods producer price inflation from steady state in period t, respectively. We outline the derivation of this expression in Appendix B.

Similarly, the export producer price inflation equation in the intermediate goods sector becomes:

$$\dot{p}_t^{xvf} - \dot{p}^{fss} = \frac{\sigma^{xb}-1}{\chi^{xv}(1+\beta\epsilon^{xv})}r\hat{m}c_t^{xv} + \frac{\beta}{1+\beta\epsilon^{xv}}(E_t\dot{p}_{t+1}^{xvf} - \dot{p}_t^{fss}) + \frac{\epsilon^{xv}}{1+\beta\epsilon^{xv}}(\dot{p}_{t-1}^{xvf} - \dot{p}_t^{fss}), \quad \textbf{(95)}$$

where $r\hat{m}c_t^{xv} = \hat{m}c_t - \hat{pxv}_t$ and $\dot{p}_t^{xvf} - \dot{p}^{fss} = (\dot{p}_t^f - \dot{p}^{fss}) - (\dot{p}_t - \dot{p}^{ss}) + (\dot{p}_t^{xv} - \dot{p}^{ss}) + \hat{q}_t - \hat{q}_{t-1}.$[9]

The import price inflation equation is given by:

$$\dot{p}_t^m - \dot{p}_t^{ss} = \frac{[1-\beta(1-\psi^m)]\psi^m}{(1-\psi^m)(1+\beta\epsilon^m)}r\hat{m}c_t^f + \frac{\beta}{1+\beta\epsilon^m}(E_t\dot{p}_{t+1}^m - \dot{p}_t^{ss}) + \frac{\epsilon^m}{1+\beta\epsilon^m}(\dot{p}_{t-1}^m - \dot{p}_t^{ss}), \quad \textbf{(96)}$$

where $r\hat{m}c_t^f = \hat{pxf}_t - \hat{q}_t - \hat{pm}_t$ is the percentage deviation of the foreign real marginal cost of production from its steady state in period t. We outline the derivation of the expression for import price inflation in Appendix C.

---

[9]Notice that here we made use of the fact that $pxvf_t = pxv_t q_t$.

Finally, inflation in the final domestic and export goods sector will depend on a weighted average of producer price and import price inflation as shown below:

$$\hat{ph}_t = \kappa^{hv}\frac{phv^{ss}}{ph^{ss}}\hat{phv}_t + (1-\kappa^{hv})\frac{pm^{ss}}{ph^{ss}}\hat{pm}_t$$

$$\hat{px}_t = \kappa^{xv}\frac{pxv^{ss}}{px^{ss}}\hat{pxv}_t + (1-\kappa^{xv})\frac{pm^{ss}}{px^{ss}}\hat{pm}_t.$$

The rigidities in wage-setting lead to the following equation for nominal wage inflation:

$$\dot{w}_t-\dot{p}^{ss} = -\frac{\psi^w[1-\beta(1-\psi^w)]}{(1-\psi^w)(1+\beta\epsilon^w)}(1+\frac{\sigma^w}{\sigma^h})^{-1}(\hat{w}_t-\hat{mrs}_t)+\frac{\beta}{(1+\beta\epsilon^w)}(E_t\dot{w}_{t+1}-\dot{p}^{ss})+\frac{\epsilon^w}{(1+\beta\epsilon^w)}(\dot{w}_{t-1}-\dot{p}^{ss}).$$
$$\tag{97}$$

We outline the derivation of this expression in Appendix A.

The equations for firms' demand for labour and capital services are given by:

$$\hat{h}_t = -\gamma_w^h\hat{w}_t + \gamma_r^h\hat{r}_t + \hat{y}_t^v + (\sigma^y-1)t\hat{fp}_t$$

$$\hat{k}_t^s = -\gamma_r^k\hat{r}_t + \gamma_w^k\hat{w}_t + \hat{y}_t^v + (\sigma^y-1)t\hat{fp}_t,$$

where[10]

$$\gamma_w^h = \sigma^y[1-\frac{\partial\hat{mc}_t}{\partial\hat{w}_t}] > 0$$

$$\gamma_r^h = \sigma^y[\frac{\partial\hat{mc}_t}{\partial\hat{r}_t}] > 0$$

$$\gamma_w^k = \sigma^y[1-\frac{\partial\hat{mc}_t}{\partial\hat{r}_t}] > 0$$

$$\gamma_r^k = \sigma^y[\frac{\partial\hat{mc}_t}{\partial\hat{r}_t}] > 0.$$

As the equations show, the demand for labour (capital services) depends negatively (positively) on the real wage rate of labour, positively (negatively) on the rental rate of capital, positively on output (with a unit elasticity), and negatively on the total productivity shock. The elasticity of labour demand (capital services) with respect to wage and rental rate reflects the elasticity of substitution between labour and capital and the elasticity of marginal cost with respect to wage and rental rate, respectively.[11]

---

[10]The log-linearised equation for marginal cost is
$\hat{mc}_t = \frac{(1-\alpha)^{\sigma^y}(w^{ss})^{1-\sigma^y}}{(1-\alpha)^{\sigma^y}(w^{ss})^{1-\sigma^y}+\alpha^{\sigma^y}(r^{ss})^{1-\sigma^y}}\hat{w}_t + \frac{\alpha^{\sigma^y}(r^{ss})^{1-\sigma^y}}{(1-\alpha)^{\sigma^y}(w^{ss})^{1-\sigma^y}+\alpha^{\sigma^y}(r^{ss})^{1-\sigma^y}}\hat{r}_t$, where w^{ss} and r^{ss} are the steady-state wage and rental rate.

[11]Note that $\hat{k}_t^s = \hat{z}_t + \hat{k}_{t-1}$.

The equation for the net foreign asset accumulation is given by:

$$
\begin{aligned}
\hat{nfa}_t &= \frac{1}{\beta} nfa^{ss} \hat{RF}_{t-1} + \frac{1}{\beta}[\hat{nfa}_{t-1} + nfa^{ss}\hat{q}_{t-1} - nfa^{ss}\dot{p}_t^f - nfa^{ss}\hat{q}_t] \\
&+ px^{ss}y^{x,ss}(\hat{px}_t + \hat{y}_t^x) - pm^{ss}(cm^{ss} + mi^{h,ss} + mi^{x,ss})\hat{pm}_t - pm^{ss}cm^{ss}\hat{cm}_t \\
&- pm^{ss}mi^{h,ss}\hat{mi}_t^x - pm^{ss}mi^{x,ss}\hat{mi}_t^x,
\end{aligned}
$$

where $RF_{t-1} = 1 + rf_{t-1}$ is the nominal gross interest rate in the rest of the world and $\hat{nfa}_t \cong nfa_t - nfa^{ss}$.

$$
\begin{aligned}
\underbrace{\hat{nfa}_t - nfa^{ss}[\hat{q}_{t-1} - \hat{q}_t - \dot{p}_t^f + \hat{nfa}_{t-1}]}_{-\hat{ca}_t} &= \frac{1-\beta}{\beta}nfa^{ss}[\hat{rf}_{t-1} + \hat{q}_{t-1} - \hat{q}_t - \dot{p}_t^f + \hat{nfa}_{t-1}] \\
&+ px^{ss}y^{x,ss}(\hat{px}_t + \hat{y}_t^x) - pm^{ss}(cm^{ss} + mi^{h,ss} + mi^{x,ss})\hat{pm}_t \\
&- pm^{ss}cm^{ss}\hat{cm}_t - pm^{ss}mi^{h,ss}\hat{mi}_t^x - pm^{ss}mi^{x,ss}\hat{mi}_t^x,
\end{aligned}
$$

where the left-hand side is the capital account (in opposite sign) and the right-hand side is the current account.[12] In the model, movements in the real exchange rate ensure that this expression holds in equilibrium.

The equation for the real exchange rate is given by:

$$
\hat{q}_{t+1} - \hat{q}_t + \chi^{bf}\hat{nfa}_t = \hat{RRF}_t - \hat{RRG}_t,
$$

where $RRF_t = 1 + rrf_t$ and $RRG_t = 1 + rrg_t$ are the gross real interest rates at Home and in the rest of the world. This modified version of the real UIP condition says that, for agents to be indifferent between domestic and foreign bonds, the real interest rate differentials must be equal to the real depreciation adjusted for the cost associated with holding bonds. When $\chi^{bf} = 0$ this equation collapses to the standard real UIP condition.

The total consumption expenditure of households is given by:

$$
\hat{c}_t = \frac{ch^{ss}ph^{ss}}{c^{ss}}(\hat{ph}_t + \hat{ch}_t) + \frac{cm^{ss}pm^{ss}}{c^{ss}}(\hat{pm}_t + \hat{cm}_t),
$$

where

$$
\hat{ch}_t - \hat{cm}_t = -\sigma^m(\hat{ph}_t - \hat{pm}_t).
$$

The imports of intermediate goods that enter the production of domestic and export final goods are given by:

$$
\begin{aligned}
\hat{mi}_t^h &= \hat{y}_t^{hv} = \hat{y}_t^h \\
\hat{mi}_t^x &= \hat{y}_t^{xv} = \hat{y}_t^x.
\end{aligned}
$$

---

[12]Note that $\hat{RF}_{t-1} = \frac{rf^{ss}}{1+rf^{ss}}\hat{rf}_{t-1}$.

The goods market equilibrium is given by:

$$
\begin{aligned}
\hat{y}_t^v &= t\hat{f}p_t + \frac{(1-\alpha)(h^{ss})^{\frac{\sigma^y-1}{\sigma^y}}}{[(1-\alpha)(h^{ss})^{\frac{\sigma^y-1}{\sigma^y}} + \alpha(k^{ss})^{\frac{\sigma^y-1}{\sigma^y}}]}\hat{h}_t + \frac{\alpha(k^{ss})^{\frac{\sigma^y-1}{\sigma^y}}}{[(1-\alpha)(h^{ss})^{\frac{\sigma^y-1}{\sigma^y}} + \alpha(k^{ss})^{\frac{\sigma^y-1}{\sigma^y}}]}\hat{k}_t \\
&= \frac{\kappa^{hv}ch^{ss}}{y^{v,ss}}\hat{ch}_t + \frac{\kappa^{hv}I^{ss}}{y^{v,ss}}\hat{I}_t + \frac{\kappa^{hv}g^{ss}}{y^{v,ss}}\hat{g}_t + \frac{\kappa^{xv}x^{ss}}{y^{v,ss}}\hat{x}_t \\
&= \hat{y}_t^{xv} + \hat{y}_t^{hv}.
\end{aligned}
$$

The export market clearing conditions is given by:

$$
\hat{y}_t^x = \hat{x}_t,
$$

where the demand for domestic exports is given by:

$$
\hat{x}_t = \hat{cf}_t - \sigma^x(\hat{q}_t + \hat{px}_t - \hat{pxf}_t).
$$

We specify a VAR (vector autoregressive) process for the foreign variables that enter our model. Specifically, we estimate a VAR in world demand, $\hat{cf}_t$, world nominal interest rates, $\hat{rf}_t$, world inflation, $\dot{p}_t^f$, and world relative export prices, $\hat{pxf}_t$.

The monetary policy reaction function is given by:

$$
\hat{rg}_t = \theta^{rg}\hat{rg}_{t-1} + (1-\theta^{rg})[\theta^p(\dot{p}_t - \dot{p}^{ss}) + \theta^y(\hat{y}_t^v - t\hat{f}p_t)] + \sigma_{mp}\eta_t^{mp}.
$$

The monetary authority responds to the deviation of annual inflation from the target and to (a crude measure of) the output gap. The parameter θ^{rg} captures the degree of interest rate smoothing. η^{mp} is the i.i.d. normal error term with zero mean and unit variance.

Finally, the model is closed by including the Fisher parity condition to link nominal and real interest rates:

$$
\hat{rg}_t = \hat{rrg}_t + \frac{1}{1-\beta}(\dot{p}_{t+1} - \dot{p}^{ss}).
$$

Appendix A: The derivation of wage Phillips curve in the Calvo model

For details refer to Section 2.1.3. Households set (nominal) wages in staggered contracts as outlined in Calvo (1983). The optimal wage rate households who reset wages can renegotiate is given by equation **(25)**. The evolution of the wage index is shown in equation **(27)**. The log-linearised form of the optimal real wage rate yields:

$$(1 + \frac{\sigma^w}{\sigma^h})\hat{R}_t^w + \hat{w}_t = \hat{\Xi}_t^w - \hat{\Theta}_t^w, \tag{A-1}$$

where

$$
\begin{aligned}
\hat{\Xi}_t^w &= [1 - \beta(1 - \psi^w)]\frac{\sigma^h + 1}{\sigma^h}\hat{h}_t - \beta(1 - \psi^w)\{\sigma^w(\frac{\sigma^h + 1}{\sigma^h})[(1 - \epsilon^w)\dot{p}^{ss} + \epsilon^w\dot{w}_t - \dot{w}_{t+1}]\} \\
&\quad + \beta(1 - \psi^w)\hat{\Xi}_{t+1}^w \\
\hat{\Theta}_t^w &= [1 - \beta(1 - \psi^w)](\hat{U}_{c,t} + \hat{h}_t) - \beta(1 - \psi^w)\{\sigma^w[(1 - \epsilon^w)\dot{p}^{ss} + \epsilon^w\dot{w}_t - \dot{w}_{t+1}] \\
&\quad - [(1 - \epsilon^w)\dot{p}^{ss} + \epsilon^w\dot{w}_t - \dot{p}_{t+1}]\} + \beta(1 - \psi^w)\hat{\Theta}_{t+1}^w.
\end{aligned}
$$

Substituting in for $\hat{\Xi}_t^w$ and $\hat{\Theta}_t^w$, equation **(A-1)** becomes:

$$
\begin{aligned}
(1 + \frac{\sigma^w}{\sigma^h})\hat{R}_t^w + \hat{w}_t = {}& [1 - \beta(1 - \psi^w)]\hat{mrs}_t - \beta(1 - \psi^w)\frac{\sigma^w}{\sigma^h}[(1 - \epsilon^w)\dot{p}^{ss} + \epsilon^w\dot{w}_t - \dot{w}_{t+1}] \\
& -\beta(1 - \psi^w)[(1 - \epsilon^w)\dot{p}^{ss} + \epsilon^w\dot{w}_t - \dot{p}_{t+1}] + \beta(1 - \psi^w)[(1 + \frac{\sigma^w}{\sigma^h})\hat{R}_{t+1}^w + \hat{w}_{t+1}],
\end{aligned}
$$

where $\hat{mrs}_t = \frac{1}{\sigma^h}\hat{h}_t - \hat{U}_{c,t}$. Next, we log-linearise the wage index to obtain:

$$\hat{R}_t^w \equiv \hat{\tilde{w}}_t - \hat{w}_t = \frac{\psi^w - 1}{\psi^w}[(1 - \epsilon^w)\dot{p}^{ss} + \epsilon^w\dot{w}_{t-1} - \dot{w}_t].$$

Noting that

$$\hat{w}_{t+1} = \hat{w}_t + (\dot{w}_{t+1} - \dot{p}^{ss}) - (\dot{p}_{t+1} - \dot{p}^{ss}),$$

and substituting in for $\hat{R}_t^w$ and $\hat{R}_{t+1}^w$ equation **(A-1)** becomes:

$$\dot{w}_t - \dot{p}^{ss} = \frac{\psi^w[1 - \beta(1 - \psi^w)]}{(1 - \psi^w)(1 + \beta\epsilon^w)}(1 + \frac{\sigma^w}{\sigma^h})^{-1}(\hat{mrs}_t - \hat{w}_t) + \frac{\beta}{(1 + \beta\epsilon^w)}(\dot{w}_{t+1} - \dot{p}^{ss}) + \frac{\epsilon^w}{(1 + \beta\epsilon^w)}(\dot{w}_{t-1} - \dot{p}^{ss}).$$

Appendix B: The derivation of Phillips curve in the Rotemberg model

For details refer to Section 2.2.1. Home producers supplying to the domestic market face Rotemberg-type costs in adjusting nominal prices as shown in equation **(53)**. This gives rise to an optimal pricing rule in which the firm charges an endogenous mark-up over marginal costs. This optimal price and the mark-up are given in equations **(57)** and **(58)**, respectively. Log-linearising the pricing equation gives us:

$$\hat{phv}_t = \hat{\psi}_t^{hv} + \hat{mc}_t,$$

or

$$\hat{\psi}_t^{hv} = -\hat{rmc}_t^{hv},$$

where $rmc_t^{hv} = \frac{mc_t}{phv_t}$ is the real marginal cost. Log deviation of ψ_t^{hv} from its steady-state value satisfies:

$$\psi^{hv}\hat{\psi}^{hv} = \frac{-\sigma^{hb}\chi^{hv}}{(\sigma^{hb}-1)^2}(\hat{\xi}_t^{hv} - \beta\hat{\xi}_{t+1}^{hv}),$$

where

$$\hat{\xi}_t^{hv} = \dot{p}_t^{hv} - (1-\epsilon^{hv})\dot{p}^{ss} - \epsilon^{hv}\dot{p}_{t-1}^{hv}$$

$$\hat{\xi}_{t+1}^{hv} = \dot{p}_{t+1}^{hv} - (1-\epsilon^{hv})\dot{p}^{ss} - \epsilon^{hv}\dot{p}_t^{hv}.$$

Further, given that the steady-state mark-up satisfies $\psi^{hv} = \frac{\sigma^{hb}}{\sigma^{hb}-1}$ we have:

$$\hat{\psi}^{hv} = \frac{-\chi^{hv}}{(\sigma^{hb}-1)}[(1+\beta\epsilon^{hv})(\dot{p}_t^{hv} - \dot{p}^{ss}) - \beta(\dot{p}_{t+1}^{hv} - \dot{p}^{ss}) - \epsilon^{hv}(\dot{p}_{t-1}^{hv} - \dot{p}^{ss})].$$

Then, using the fact that the log deviation of mark-up is equal to the negative of the log deviation of real marginal costs we can arrange the above equation to give:

$$\dot{p}_t^{hv} - \dot{p}^{ss} = \frac{\sigma^{hb}-1}{\chi^{hv}(1+\beta\epsilon^{hv})}\hat{rmc}_t^{hv} + \frac{\beta}{1+\beta\epsilon^{hv}}(\dot{p}_{t+1}^{hv} - \dot{p}_t^{ss}) + \frac{\epsilon^{hv}}{1+\beta\epsilon^{hv}}(\dot{p}_{t-1}^{hv} - \dot{p}_t^{ss}).$$

Appendix C: The derivation of Phillips curve in the Calvo model

For details refer to Section 2.6. Import prices are set in Home's currency and in staggered contracts as in Calvo (1983). This gives rise to an optimal pricing rule shown in equation **(73)** and an import price index shown in equation **(74)**. The log-linearised form of the first-order condition that governs the optimal import price yields:

$$\hat{R}_t^m \equiv \widehat{\widetilde{PM}}_t - \hat{PM}_t = \hat{\Xi}_t^m - \hat{\Theta}_t^m,$$

where we use R^m to denote the relative import price of the firm and where:

$$\hat{\Xi}_t^m = [1 - \beta(1 - \psi^{pm})](r\hat{mc}_t^f + \hat{ym}_t) + \beta(1 - \psi^{pm})\{(\sigma^{mb} + 1)[\dot{p}_{t+1}^m + (\epsilon^m - 1)\dot{p}^{ss} - \epsilon^m \dot{p}_t^m] + \hat{\Xi}_{t+1}^m\}$$

$$\hat{\Theta}_t^m = [1 - \beta(1 - \psi^{pm})](\hat{ym}_t) + \beta(1 - \psi^{pm})\{\sigma^{mb}[\dot{p}_{t+1}^m + (\epsilon^m - 1)\dot{p}^{ss} - \epsilon^m \dot{p}_t^m] + \hat{\Theta}_{t+1}^m\}.$$

Subtracting $\hat{\Theta}_t^m$ from $\hat{\Xi}_t^m$ we obtain:

$$\hat{\Xi}_t^m - \hat{\Theta}_t^m = [1 - \beta(1 - \psi^{pm})]r\hat{mc}_t^f + \beta(1 - \psi^{pm})[\dot{p}_{t+1}^m + (\epsilon^m - 1)\dot{p}^{ss} - \epsilon^m \dot{p}_t^m + (\hat{\Xi}_{t+1}^m - \hat{\Theta}_{t+1}^m)].$$

This implies:

$$\hat{R}_t^m = [1 - \beta(1 - \psi^{pm})]r\hat{mc}_t^f + \beta(1 - \psi^{pm})[\dot{p}_{t+1}^m + (\epsilon^m - 1)\dot{p}^{ss} - \epsilon^m \dot{p}_t^m] + \beta(1 - \psi^{pm})\hat{R}_{t+1}^m. \quad \textbf{(C-1)}$$

Log-linearising the import price index we obtain:

$$\hat{R}_t^m = \frac{\psi^{pm} - 1}{\psi^{pm}}[(1 - \epsilon^m)\dot{p}^{ss} + \epsilon^m \dot{p}_{t-1}^m - \dot{p}_t^m]. \quad \textbf{(C-2)}$$

Substituting **(C-2)** in **(C-1)** we have:

$$\dot{p}_t^m - \dot{p}_t^{ss} = \frac{[1 - \beta(1 - \psi^{pm})]\psi^{pm}}{(1 - \psi^{pm})(1 + \beta\epsilon^m)}r\hat{mc}_t^f + \frac{\beta}{1 + \beta\epsilon^m}(\dot{p}_{t+1}^m - \dot{p}_t^{ss}) + \frac{\epsilon^m}{1 + \beta\epsilon^m}(\dot{p}_{t-1}^m - \dot{p}_t^{ss}).$$

Finally, the log-linearised form of rmc_t^f is given by:

$$r\hat{mc}_t^f = p\hat{x}f_t - \hat{q}_t - \hat{pm}_t.$$

Table A: Model equations that determine the stationary and symmetric equilibrium

PPI

$$phv_t = \Psi_t^{hv} mc_t$$

$$pxv_t = \Psi_t^{xv} mc_t$$

$$pxvf_t = q_t pxv_t$$

$$ph_t = \kappa^{hv} phv_t + (1-\kappa^{hv})pm_t$$

$$px_t = \kappa^{xv} pxv_t + (1-\kappa^{xv})pm_t$$

MP

$$pm_t^{1-\sigma^{mb}} = (1-\psi^{pm})(\xi_{t-1,t}^m)(\frac{pm_{t-1}}{1+\dot p_t})^{1-\sigma^{mb}} + \psi^{pm}(\widetilde{pm}_t)^{1-\sigma^{mb}}$$

$$\frac{\widetilde{pm}_t}{\widetilde{pm}_t} = pm_t \frac{\sigma^{mb}}{\sigma^{mb}-1}\frac{\Xi_t^m}{\Theta_t^m}$$

$$\Xi_t^m = rmc_t^f cm_t + \beta(1-\psi^{pm}) E_t\left(\frac{\xi_{t,t+1}^m pm_t}{(1+\dot p_{t+1})pm_{t+1}}\right)^{-(\sigma^{mb}+1)}\Xi_{t+1}^m$$

$$\Theta_t^m = cm_t + \beta(1-\psi^{pm}) E_t\left(\frac{\xi_{t,t+1}^m pm_t}{(1+\dot p_{t+1})pm_{t+1}}\right)^{-\sigma^{mb}}\Theta_{t+1}^m$$

$$\xi_{t-1,t}^m = (1+\dot p^{ss})^{1-\epsilon^m}[(1+\dot p_{t-1})\frac{pm_{t-1}}{pm_{t-2}}]^{\epsilon^m}$$

$$rmc_t^f = \frac{px_t f_t}{q_t pm_t}$$

MU

$$\Psi_t^{hv} = \sigma^{hb}\{(\sigma^{hb}-1) + \chi^{hv}\Upsilon_t^{hv}\}^{-1}$$

$$\Upsilon_t^{hv} \cong \xi_t^{hv}(\xi_t^{hv}+1) - (1+rrg_t)^{-1}E_t\{\frac{phv_{t+1}}{phv_t}\frac{y_{t+1}^{hv}}{y_t^{hv}}\xi_{t+1}^{hv}(\xi_{t+1}^{hv}+1)\}$$

$$\xi_t^{hv} \equiv \frac{(1+\dot p_t)\frac{phv_t}{phv_{t-1}}}{(1+\dot p^{ss})^{1-\epsilon^{hv}}[(1+\dot p_{t-1})\frac{phv_{t-1}}{phv_{t-2}}]^{\epsilon^{hv}}} - 1$$

$$\Psi_t^{xv} = \sigma^{xb}\{(\sigma^{xb}-1) + \chi^{xv}\Upsilon_t^{xv}\}^{-1}$$

$$\Upsilon_t^{xv} \cong \xi_t^{xv}(\xi_t^{xv}+1) - (1+rrg_t)^{-1}E_t\{\frac{pxv_{t+1}}{pxv_t}\frac{y_{t+1}^{xv}}{y_t^{xv}}\xi_{t+1}^{xv}(\xi_{t+1}^{xv}+1)\}$$

$$\xi_t^{xv} \equiv \frac{(1+\dot p_t^f)\frac{pxvf_t}{pxvf_{t-1}}}{(1+\dot p^{ss})^{1-\epsilon^{xv}}[(1+\dot p_{t-1}^f)\frac{pxvf_{t-1}}{pxvf_{t-2}}]^{\epsilon^{xv}}} - 1$$

DMD

$$c_t = ph_t ch_t + pm_t cm_t$$

$$cm_t = (\psi^m)\sigma^m (pm_t)^{-\sigma^m}\frac{c_t}{(\kappa^c)^{1-\sigma^m}}$$

$$ch_t = (1-\psi^m)\sigma^m (ph_t)^{-\sigma^m}\frac{c_t}{(\kappa^c)^{1-\sigma^m}}$$

$$I_t = k_t - (1-\delta)k_{t-1} + \chi^z \frac{k_{t-1}}{1+\sigma^z}[z_t^{1+\sigma^z}-1] + \frac{\chi^k}{2}\frac{[k_t-k_{t-1}]^2}{k_{t-1}}$$

$$x_t = \kappa^x (\frac{q_t px_t}{px_t f_t})^{-\sigma^x} cf_t$$

SUP

$$y_t^v = tfp_t[(1-\alpha)(h_t)^{\frac{\sigma^y-1}{\sigma^y}} + \alpha(k_t^s)^{\frac{\sigma^y-1}{\sigma^y}}]^{\frac{\sigma^y}{\sigma^y-1}}$$

$$y_t^h = \frac{mi_t^h}{\kappa^{hv}}\frac{y_t^{hv}}{1-\kappa^{hv}}$$

$$y_t^x = \frac{mi_t^x}{\kappa^{xv}}\frac{y_t^{xv}}{1-\kappa^{xv}}$$

MUC

$$U_{c,t} = (c_{t-1})^{-\psi^{hab}} [\frac{c_t}{c_{t-1}^{\psi^{hab}}}]^{\frac{-1}{\sigma^c}}$$

FOCs

$$\beta E_t U_{c,t+1} = U_{c,t}(1+rrg_t)^{-1}$$

$$mon_t = \kappa^{mon}[U_{c,t}(\frac{rg_t}{1+rg_t})]^{-\sigma^c}$$

$$ph_t(1+\chi^k\frac{k_{t-1}}{k_{t-2}})^{\epsilon^k}k_{t-1})$$

$$\cong \frac{1}{1+rrg_t}E_t\{r_{t+1}z_{t+1} + ph_{t+1}(1-\delta-\chi^z\frac{z_{t+1}^{1+\sigma^z}-1}{1+\sigma^z} + \chi^k(\frac{k_t}{k_{t-1}})^{\epsilon^k}\frac{k_{t+1}-(\frac{k_t}{k_{t-1}})^{\epsilon^k}k_t}{k_t}\}$$

$$r_t = ph_t \chi^z z_t^{\sigma^z}$$

$$w_t^{1-\sigma^w} = (1-\psi^w)[\xi_{t-1,t}^w\frac{w_{t-1}}{1+\dot p_t}]^{1-\sigma^w} + \psi^w(\widetilde w_t)^{1-\sigma^w}$$

$$\widetilde w_t^{-1+\frac{\sigma^w}{\sigma^h}} = w_t^{\frac{\sigma^w}{\sigma^h}}(\kappa^h)^{-\frac{\sigma^w}{\sigma^h}}\frac{\sigma^w}{\sigma^w-1}\frac{\Xi_t^w}{\Theta_t^w}$$

$$\Xi_t^w = h_t^{1+\frac{\sigma^h+1}{\sigma^h}} + \beta(1-\psi^w)E_t[\frac{\xi_{t,t+1}^w w_t}{(1+\dot p_{t+1})w_{t+1}}]^{-\sigma^w(\frac{\sigma^h+1}{\sigma^h})}\Xi_{t+1}^w$$

$$\Theta_t^w = U_{c,t}h_t + \beta(1-\psi^w)E_t(1+\dot p_{t+1})\xi_{t,t+1}^w[\frac{\xi_{t,t+1}^w w_t}{(1+\dot p_{t+1})w_{t+1}}]^{-\sigma^w}\Theta_{t+1}^w$$

$$\xi_{t-1,t}^w = (1+\dot p^{ss})^{1-\epsilon^w}[(1+\dot p_{t-1})\frac{w_{t-1}}{w_{t-2}}]^{\epsilon^w}$$

$$h_t = (\frac{w_t}{1-\alpha})^{-\sigma^y}mc_t^{\sigma^y}tfp_t^{\sigma^y-1}y_t^v$$

$$k_t^s = (\frac{r_t}{\alpha})^{-\sigma^y}mc_t^{\sigma^y}tfp_t^{\sigma^y-1}y_t^v$$

UR

$$k_t^s = z_t k_{t-1}$$

RUIP

$$E_t\frac{q_{t+1}}{q_t}[1+\chi^{bf}(\frac{bf_t}{q_t}-nfa^{ss})] \cong \frac{1+rrf_t}{1+rrg_t}$$

NFA

$$\frac{bf_t}{q_t} = \frac{1+rf_{t-1}}{1+\dot p_t^f}\frac{bf_{t-1}}{q_t} + px_t y_t^x - pm_t(cm_t + mi_t^h + mi_t^x)$$

MPR

$$\frac{1+rg_t}{1+rg^{ss}} = (\frac{1+rg_{t-1}}{1+rg^{ss}})^{\theta^{rg}}\{(\frac{1+\dot p_t}{1+\dot p^{ss}})^{\theta^p}(\frac{y_t}{y^{v,ss}})^{\theta^y}]\}^{1-\theta^{rg}}$$

GMC

$$y_t^v = \kappa^{hv}y_t^h + \kappa^{xv}y_t^x$$

$$y_t^h = ch_t + I_t + g_t$$

$$y_t^x = x_t$$

FPC

$$E_t(1+\dot p_{t+1})(1+rrg_t) \equiv (1+rg_t)$$

INFs

$$1+\dot p_t^{hv} = (1+\dot p_t)\frac{phv_t}{phv_{t-1}}$$

$$1+\dot p_t^{xvf} = (1+\dot p_t^f)\frac{pxvf_t}{pxvf_{t-1}}$$

$$1+\dot p_t^m = (1+\dot p_t)\frac{pm_t}{pm_{t-1}}$$

$$1+\dot p_t^w = (1+\dot p_t)\frac{w_t}{w_{t-1}}$$

Note. Abbreviations are used for Consumer Price Index (CPI), Producer Price Index (PPI), Import Price (MP), Mark-Up (MU), Marginal Utility of Consumption (MUC), First-Order Conditions (FOCs), Supply (SUP), Demand (DMD), Mark-Up (MU), Marginal Utility of Consumption (MUC), First-Order Conditions (FOCs), Utilisation Rate (UR), Real UIP (RUIP), Net Foreign Assets (NFA), Monetary Policy Rule (MPR), Goods Market Clearing (GMC), Fisher Parity Condition (FPC), and Inflation Equations (INFs).

Table B: Model equations at the steady state

PPI	$phv^{ss} = \Psi^{hv,ss} mc^{v,ss}$ $pxv^{ss} = \Psi^{xv,ss} mc^{v,ss}$ $pxvf^{ss} = q^{ss} pxv^{ss}$ $ph^{ss} = \kappa^{hv} phv^{ss} + (1 - \kappa^{hv}) pm^{ss}$ $px^{ss} = \kappa^{xv} \frac{pxv^{ss}}{q^{ss}} + (1 - \kappa^{xv}) pm^{ss}$
MP	$pm^{ss} = \frac{\sigma^{mb}}{\sigma^{mb}-1} \frac{pxf^{ss}}{q^{ss}}$
MU	$\Psi^{hv,ss} = \frac{\sigma^{hb}}{\sigma^{hb}-1}$ $\Psi^{xv,ss} = \frac{\sigma^{xb}}{\sigma^{hb}-1}$
DMD	$c^{ss} = ph^{ss} ch^{ss} + pm^{ss} cm^{ss}$ $cm^{ss} = (\psi^m)^{\sigma^m} (pm^{ss})^{-\sigma^m} \frac{c^{ss}}{(\kappa^c)^{1-\sigma^m}}$ $ch^{ss} = (1-\psi^m)^{\sigma^m} (ph^{ss})^{-\sigma^m} \frac{c^{ss}}{(\kappa^c)^{1-\sigma^m}}$ $I^{ss} = \delta k^{ss}$ $x^{ss} = \kappa^x \frac{q^{ss} px^{ss}}{pxf^{ss}} cf^{ss}$
SUP	$y^{v,ss} = tfp^{ss}\left[(1-\alpha)(h^{ss})^{\frac{\sigma^y-1}{\sigma^y}} + \alpha(k^{ss})^{\frac{\sigma^y-1}{\sigma^y}}\right]^{\frac{\sigma^y}{\sigma^y-1}}$ $y^{h,ss} = \frac{y^{hv,ss}}{\kappa^{hv}} = \frac{mi^{h,ss}}{1-\kappa^{hv}}$ $y^{x,ss} = \frac{y^{xv,ss}}{\kappa^{xv}} = \frac{mi^{x,ss}}{1-\kappa^{xv}}$

MUC	$U_c^{ss} = [c^{ss}]^{\frac{(\psi^{hab}-1-\psi^{hab}\sigma^c)}{\sigma^c}}$
FOCs	$\beta = (1+rrg^{ss})^{-1}$ $mon^{ss} = \kappa^{mon}(U_c^{ss}(1-\beta))^{-\sigma^c}$ $ph^{ss} = \frac{1}{1+rrg^{ss}}\{r^{ss} + ph^{ss}(1-\delta)\}$ $r^{ss} = ph^{ss}\chi^z$ $w^{ss} = \frac{(\kappa^h)^{-\frac{1}{\sigma^h}}\sigma^w (h^{ss})^{\frac{1}{\sigma^h}}}{\sigma^w-1} \frac{1}{U_{c,ss}}$ $h^{ss} = \left(\frac{w^{ss}}{1-\alpha}\right)^{-\sigma^y}(mc^{ss})^{\sigma^y}(tfp^{ss})^{\sigma^y-1}y^{v,ss}$ $k^{s,ss} = \left(\frac{r^{ss}}{\alpha}\right)^{-\sigma^y}(mc^{ss})^{\sigma^y}(tfp^{ss})^{\sigma^y-1}y^{v,ss}$
UR	$k^{s,ss} = z^{ss}k^{ss}$
RUIP	$1 = \frac{1+rrf^{ss}}{1+rrg^{ss}}$
NFA	$nfa^{ss} = \frac{bf^{ss}}{q^{ss}} = \frac{1}{rf^{ss}}[pm^{ss}(cm^{ss} + mi^{h,ss} + mi^{x,ss}) - px^{ss}y^{x,ss}]$
GMC	$y^{v,ss} = \kappa^{hv}y^{h,ss} + \kappa^{xv}y^{x,ss}$ $y^{h,ss} = ch^{ss} + I^{ss} + g^{ss}$ $y^{x,ss} = x^{ss}$
FPC	$1 + rrg^{ss} = 1 + rg^{ss}$
INFs	$\dot{p}^{ss} = \dot{p}^{hv,ss} = \dot{p}^{xvf,ss} = \dot{p}^{m,ss} = \dot{p}^{w,ss} = 0$

Note. Abbreviations are used for Consumer Price Index (CPI), Producer Price Index (PPI), Import Price (MP), Mark-Up (MU), Demand (DMD), Supply (SUP), Marginal Utility of Consumption (MUC), First-Order Conditions (FOCs), Utilisation Rate (UR), Real UIP (RUIP), Net Foreign Assets (NFA), Goods Market Clearing (GMC), Fisher Parity Condition (FPC), and Inflation Equations (INFs).

Table C: The log-linearised model equations

PPI

$$\hat{phv}_t = \hat{\Psi}^{hv}_t + \hat{mc}_t$$
$$\hat{pxv}_t = \hat{\Psi}^{xv}_t + \hat{mc}_t$$
$$\hat{pxvf}_t = \hat{q}_t + \hat{pxv}_t$$
$$\hat{ph}_t = \kappa^{hv}\frac{phv^{ss}}{ph^{ss}}\hat{phv}_t + (1-\kappa^{hv})\frac{pm^{ss}}{ph^{ss}}\hat{pm}_t$$
$$\hat{px}_t = \kappa^{xv}\frac{pxv^{ss}}{px^{ss}}\hat{pxv}_t + (1-\kappa^{xv})\frac{pm^{ss}}{px^{ss}}\hat{pm}_t$$

MP

$$\hat{p}^m_t - \dot{p}^{ss} = \frac{\psi^{pm}[1-\beta(1-\psi^{pm})]}{(1+\beta\epsilon^m)(1-\psi^{pm})}(\hat{pxf}_t - \hat{q}_t - \hat{pm}_t) + \frac{\beta}{1+\beta\epsilon^m}(\dot{p}^m_{t+1} - \dot{p}^{ss}) + \frac{\epsilon^m}{1+\beta\epsilon^m}(\dot{p}^m_{t-1} - \dot{p}^{ss})$$

MU

$$\hat{\Psi}^{hv}_t = -\frac{\chi^{hv}}{\sigma^{hb}-1}[[(1+\beta\epsilon^{hv})(\dot{p}^{hv}_t - \dot{p}^{ss}) - \epsilon^{hv}(\dot{p}^{hv}_{t-1} - \dot{p}^{ss}) - \beta(\dot{p}^{hv}_{t+1} - \dot{p}^{ss})]$$
$$\hat{\Psi}^{xv}_t = -\frac{\chi^{xv}}{\sigma^{xb}-1}[[(1+\beta\epsilon^{xv})(\dot{p}^{xv}_t - \dot{p}^{fss}) - \epsilon^{xv}(\dot{p}^{xv}_{t-1} - \dot{p}^{fss}) - \beta(\dot{p}^{xvf}_{t+1} - \dot{p}^{fss})]$$

DMD

$$\hat{c}_t = \frac{ch^{ss}}{c^{ss}}(\hat{ph}_t + \hat{ch}_t) + \frac{cm^{ss}pm^{ss}}{c^{ss}}(\hat{pm}_t + \hat{cm}_t)$$
$$\hat{ch}_t = \hat{c}_t - \sigma^m\hat{ph}_t$$
$$\hat{cm}_t = \hat{c}_t - \sigma^m\hat{pm}_t$$
$$\hat{\delta i}_t = \hat{k}_t - (1-\delta)\hat{k}_{t-1} + \chi^z\hat{z}_t$$
$$\hat{x}_t = \hat{cf}_t - \sigma^x(\hat{q}_t + \hat{pxf}_t - \hat{pxf}_t)$$

SUP

$$\hat{y}^v_t = t\hat{fp}_t + \frac{(1-\alpha)(h^{ss})^{\frac{\sigma^y-1}{\sigma^y}}}{[(1-\alpha)(h^{ss})^{\frac{\sigma^y-1}{\sigma^y}}+\alpha(k^{ss})^{\frac{\sigma^y-1}{\sigma^y}}]}\hat{h}_t + \frac{\alpha(k^{ss})^{\frac{\sigma^y-1}{\sigma^y}}}{[(1-\alpha)(h^{ss})^{\frac{\sigma^y-1}{\sigma^y}}+\alpha(k^{ss})^{\frac{\sigma^y-1}{\sigma^y}}]}\hat{k}^s_t$$
$$\hat{y}^h_t = \hat{y}^{hv}_t = \hat{mi}^h_t$$
$$\hat{y}^x_t = \hat{y}^{xv}_t = \hat{mi}^x_t$$

MUC

$$\hat{U}_{C,t} = \left(\frac{\psi^{hab} - \sigma^c\psi^{hab}}{\sigma^c}\right)\hat{c}_{t-1} - \left(\frac{\psi^{hab} - \sigma^c\psi^{hab}}{\sigma^c}\right)\hat{c}_{t-1} + \frac{1}{\sigma^c}\hat{c}_t$$

MRS

$$\hat{mrs}_t = \frac{1}{\sigma^h}\hat{h}_t + \hat{phv}_t - \hat{phv}_t$$

INFs

$$\hat{p}^{hv}_t = \hat{p}_t + \hat{phv}_t - \hat{phv}_{t-1}$$
$$\hat{p}^{xvf}_t = \hat{p}^f_t + \hat{pxvf}_t - \hat{pxvf}_{t-1}$$
$$\hat{p}^m_t = \hat{p}_t + \hat{pm}_t - \hat{pm}_{t-1}$$
$$\hat{p}^w_t = \hat{p}_t + \hat{w}_t - \hat{w}_{t-1}$$

FOCs

$$\hat{U}_{C,t+1} = \hat{U}_{C,t} - R\hat{R}G_t$$
$$\hat{m\hat{o}n}_t = -\sigma^c\left(\hat{U}_{C,t} + \frac{\beta}{1-\beta}R\hat{R}G_t\right)$$
$$\hat{ph}_t + \chi^k(\hat{k}_t - \hat{k}_{t-1} - \epsilon^k(\hat{k}_{t-1} - \hat{k}_{t-2})) + R\hat{R}G_t = \beta\chi^z\hat{z}_{t+1}$$
$$+\beta(1-\delta)\hat{ph}_{t+1} + \beta\chi^k(\hat{k}_{t+1} - \hat{k}_t - \epsilon^k(\hat{k}_t - \hat{k}_{t-1}))$$
$$\hat{r}_t = \hat{ph}_t + \sigma^z\hat{z}_t$$
$$\hat{w}_t - \dot{p}^{ss} = \frac{\psi^w[1-\beta(1-\psi^w)]}{(1-\psi^w)(1+\beta\epsilon^w)}\left(1+\frac{\sigma^w}{\sigma^h}\right)^{-1}(\hat{mrs}_t - \hat{w}_t) + \frac{\beta}{(1+\beta\epsilon^w)}(\dot{w}_{t+1} - \dot{p}^{ss})$$
$$+\frac{\epsilon^w}{(1+\beta\epsilon^w)}(\dot{w}_{t-1} - \dot{p}^{ss})$$
$$\hat{h}_t = -\sigma^y\hat{w}_t + \sigma^y\hat{mc}_t + \hat{y}^v_t + (\sigma^y - 1)t\hat{fp}_t$$
$$\hat{k}^s_t = -\sigma^y\hat{r}_t + \sigma^y\hat{mc}_t + \hat{y}^v_t + (\sigma^y - 1)t\hat{fp}_t$$
$$\hat{k}^s_t = \hat{z}_t + \hat{k}_{t-1}$$

RUIP

$$\hat{q}_{t+1} - \hat{q}_t + \chi^{bf}(\hat{nfa}_t - nfa^{ss}) = R\hat{R}F_t - R\hat{R}G_t$$

NFA

$$\hat{nfa}_t = \frac{1}{\beta}\hat{nfa}_{t-1} + \frac{1}{\beta}nfa^{ss}R\hat{R}F_{t-1} - \frac{1}{\beta}nfa^{ss}\dot{p}^f_t$$
$$+\frac{1}{\beta}nfa^{ss}\hat{q}_{t-1} - \frac{1}{\beta}nfa^{ss}\hat{q}_t + px^{ss}y^{x,ss}(\hat{px}_t + \hat{y}^x_t)$$
$$-pm^{ss}(cm^{ss} + mi^{h,ss} + mi^{x,ss})\hat{pm}_t - pm^{ss}cm^{ss}\hat{cm}_t$$
$$-pm^{ss}mi^{h,ss}\hat{mi}^h_t - pm^{ss}mi^{x,ss}\hat{mi}^x_t$$

MPR

$$\hat{rg}_t = \theta^{rg}\hat{rg}_{t-1} + (1-\theta^{rg})[\theta^p(\dot{p}_t - \dot{p}^{ss}) + \theta^y(\hat{y}^v_t - t\hat{fp}_t)]$$

GMC

$$\hat{y}^v_t = \theta^{rg}\frac{\kappa^{hv}ch^{ss}}{y^{v,ss}}\hat{ch}_t + \frac{\kappa^{hv}ss}{y^{v,ss}}\hat{I}_t + \frac{\kappa^{hv}g^{ss}}{y^{v,ss}}\hat{g}_t + \frac{\kappa^{xv}x^{ss}}{y^{v,ss}}\hat{x}_t$$
$$\hat{y}^v_t = \frac{y^{v,ss}}{y^{hv,ss}}\hat{y}^{hv} - \frac{y^{xv,ss}}{y^{v,ss}}\hat{y}^{xv}$$
$$\hat{y}^x_t = \hat{x}_t$$

FPC

$$R\hat{R}G_t = R\hat{R}G_t + (E_t\dot{p}_{t+1} - \dot{p}^{ss})$$

Note. [1] Abbreviations are used for Consumer Price Index (CPI), Producer Price Index (PPI), Import Price (MP), Mark-Up (MU), Demand (DMD), Supply (SUP), Marginal Utility of Consumption (MUC), First-Order Conditions (FOCs), Utilisation Rate (UR), Real UIP (RUIP), Net Foreign Assets (NFA), Monetary Policy Rule (MPR), Goods Market Clearing (GMC), Fisher Parity Condition (FPC), and Inflation Equations (INFs). [2] $\hat{RG}_t \cong (rg_t - rrg^{ss})$, $\hat{RF}_t \cong (rf_t - rf^{ss})$, $R\hat{R}F_t \cong (rrf_t - rrf^{ss})$, $R\hat{R}G_t \cong (rrg_t - rrg^{ss})$, $\hat{nfa}_t \cong nfa_t - nfa^{ss}$.

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