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Non-rational expectations and the transmission mechanism

Richard Harrison and Tim Taylor

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Non-rational expectations and the transmission mechanism

Richard Harrison⁽¹⁾ and Tim Taylor⁽²⁾

Abstract

In this paper, we compare two approaches to modelling behaviour under non-rational expectations in a benchmark New Keynesian model. The ‘Euler equation’ approach modifies the equations derived under the assumption of rational expectations by replacing the rational expectations operator with an alternative assumption about expectations formation. The ‘long-horizon’ expectations approach solves the decision rules of households and firms conditional on their expectations for future events that are outside of their control, so that spending and price-setting decisions depend on expectations extending into the distant future. Both approaches can be defended as descriptions of (distinct) forms of boundedly rational behaviour, but have different implications both for the form of the equations that govern the dynamics of the economy and the ease of deriving those equations. In this paper we construct two versions of a benchmark New Keynesian model in which non-rational expectations are modelled using the Euler equation and long-horizon approaches and show that both approaches have very similar implications for macroeconomic dynamics when departures from rational expectations are relatively small. But as expectations depart further from rationality, the two approaches can generate significantly different implications for the behaviour of key variables.

Key words: Expectations, monetary transmission mechanism.

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Summary

Models are important tools that economists use to help them understand the behaviour of the economy. Many macroeconomic models assume that the decisions of households and firms should depend on their expectation of future events. For example, a household's saving decision is likely to be influenced by an assessment of the income that is expected to be earned in the future. And the price a firm decides to set for its product is likely to depend on its view of the costs of production that it will incur over the period until it next resets its price. An important assumption for such models is how households and firms form their expectations of future earnings and costs. The dominant assumption in macroeconomics is that expectations are formed in a way that is 'rational' (or 'model consistent'). An implication is that expectations are correct on average and that the difference between expected and actual outturns are unpredictable. In other words, households and firms do not make persistent mistakes when predicting future earnings or costs.

The rational expectations assumption is a very strong one, implying that households and firms have a lot of information about the structure of the economy. This has led economists and policymakers to examine the effects of alternative 'non-rational' expectations assumptions. Relative to the benchmark assumption of rational expectations, models that include non-rational expectations face two challenges. The first is the need to specify the mechanism through which expectations are generated. The second is how to capture the way that expectations of future earnings and costs affect the decisions that households and firms make about their current savings and pricing.

This paper is concerned with the second challenge. There are two main alternatives to modelling decision making when expectations are non-rational. To see the difference between these, suppose, as an example, that a household makes a decision over how much to save and how much to spend. The decision depends on the household's expectations of future earnings: higher future earnings allow the household to borrow to finance higher spending today. There are two ways to characterise how the household decides how much to spend and save.

The first approach relies on the consumption 'Euler equation', which states that the household's



current consumption should depend on the expected level of consumption next period and the real interest rate. Other things equal, a higher real interest rate will encourage households to consume less and save more. This approach to non-rational expectations therefore assumes that household consumption is determined by the Euler equation, but with a non-rational expectation of future consumption. The second approach is to characterise the household's consumption decision in terms of the household's expectations of its entire lifetime income. Other things equal, the higher the household's expected lifetime income, the higher the household's current consumption. In this approach, consumption is therefore determined by non-rational expectations of lifetime income. We call this the 'long-horizon' approach.

Under rational expectations, the 'Euler equation' and 'long-horizon' approaches give identical answers: the household's consumption is the same in both cases. But under non-rational expectations, the predictions for consumption can be different. The purpose of this paper is to investigate how significant these differences may be. To do so, we build a model of household and firm behaviour under three assumptions: rational expectations, non-rational 'Euler equation' expectations and non-rational 'long-horizon' expectations. We then compare the behaviour of key variables for these variants of the model.

We find that when households and firms have expectations that are close to rational expectations, there is little difference between the behaviour of the 'Euler equation' and 'long-horizon' versions of the model. This means that the properties of key variables such as consumption and inflation – for example in response to a change in the interest rate set by the monetary policy maker – are very similar, regardless of the assumptions we make about expectations. But when households and firms use expectations that are further away from rational expectations, the differences between the properties of the 'Euler equation' and 'long-horizon' versions become larger. This key result has implications for economic model builders. For cases in which households and firms have expectations of future income and costs that are very different from rational expectations of those variables, the model builder should choose the approach carefully.



1 Introduction

Despite the dominance of rational expectations models in macroeconomics, policymakers and academics alike have considerable interest in the implications of alternative expectations processes for the macroeconomy.¹ For example, the rationality or otherwise of expectations is a key consideration in the analysis of monetary policy credibility and the nature of expectations formation is likely to have significant implications for the transmission mechanism of monetary policy.

Issues of this type have motivated a vast literature exploring the implications of deviations from rational expectations. Setting aside the question of how to specify the way that non-rational expectations are formed, one issue that has received some attention is how to characterise economic behaviour under non-rational expectations (see Preston (2005a) and Evans, Honkapohja and Mitra (2002)). A popular approach is to take the equations of the model derived under rational expectations – typically Euler equations describing spending and inflation – and to replace the (rational) expectations terms with alternative processes for expectations (see, for example, Evans and Honkapohja (2001)). Another approach is to solve the decision rules of households and firms conditional on their expectations for future events that are outside of their control. In this way the role played by expectations in the model is left explicit, so that spending and price-setting decisions depend on so-called ‘long-horizon expectations’ as articulated by Preston (2005b).² Specifically, households’ consumption decisions depend on the discounted sum of expected future income and real interest rates and firms’ prices depend on the discounted sum of their expected wage costs, productivity and inflation.

Both approaches can be defended as descriptions of (distinct) forms of boundedly rational behaviour: see, for example, the discussion in Branch and McGough (2006). Indeed, Evans, Honkapohja and Mitra (2003) note that the two approaches are equivalent if additional informational assumptions are applied to the ‘long-horizon expectations’ approach. We prefer the description implied by the long-horizon expectations approach on the grounds of consistency with the underlying microfoundations (precisely because it does not require the additional informational assumptions detailed by Evans *et al* (2003)). But adopting this approach comes at

¹See for example Bernanke (2007).

²This type of approach dates back at least to Townsend (1983) and Sargent (1991).



the cost of a loss of tractability as is evident from the derivation of our model in Section 2.3. A practical question is therefore whether the choice of approach has significant differences for the properties of the transmission mechanism.³

To investigate the potential importance of the differences between them, we examine the extent to which the two approaches differ as expectations depart from rationality. We do so in two ways. First, we use relative entropy to measure the distance between the probability distributions for the endogenous variables generated by the two versions of the model. And second, we compare the impulse responses generated by the two versions. We find that, when departures from rational expectations are small, both approaches have very similar implications for macroeconomic variables. But as expectations depart further from rationality, the two approaches can generate quite different implications for the behaviour of key variables. While deriving a general result for all classes for forward-looking models is well beyond the scope of this paper, a wide range of sensitivity analysis suggests that the discrepancy between the two approaches to incorporating non-rational expectations generally increases as the deviation from rational expectations increases.

2 The model

In this section, we set out a baseline New Keynesian model and present the solution under alternative assumptions about expectations. A more detailed derivation is presented in Appendix A. So here we focus on the maximisation problems of the agents in the model and the associated log-linearised decision rules. We begin with the familiar rational expectations benchmark in Section 2.1. We then consider the ‘Euler equation’ approach to incorporating non-rational expectations in Section 2.2 and the ‘long-horizon’ approach in Section 2.3. In both cases, many of the equations carry over from the rational expectations version, so we focus discussion on the decision rules of households and firms.

³Preston (2005b) argues that the choice of approach has important implications for the conditions that ensure that the rational expectations equilibrium can be learned by agents using recursive least squares estimation. This result takes into account the dynamic interaction between the structure of the (alternative) models and the learning behaviour of the private sector. We abstract from such interaction here.

2.1 The rational expectations benchmark

Here we describe the economic environment faced by agents in our model and characterise optimal behaviour and equilibrium under the assumption that expectations are rational. We consider households, firms and government in turn.

2.1.1 Households

There is a continuum of households of unit mass, indexed by $i \in (0, 1)$. We assume that household i solves:

$$\max E_{t-1} \sum_{s=t}^{\infty} \beta^{s-t} \left[\frac{c_{i,s}^{1-\sigma} - 1}{1-\sigma} - \chi \frac{h_{i,s}^{1+\gamma}}{1+\gamma} \right]$$

where c is consumption and h is hours worked and the parameters σ and γ are both strictly positive.⁴ We use E_t to denote the mathematical expectations operator conditional on the information available to the household at date t . We impose the assumption that when making decisions about consumption at date t and beyond, the household has access to data up to and including date $t - 1$. This timing assumption is comparable with the informational restrictions that apply to agents with non-rational expectations analysed in Sections 2.2 and 2.3.

More specifically, we assume that at the beginning of the period, the household forms a consumption plan before splitting its activities between a ‘shopper’ and a ‘worker’. The shopper enters the goods market and purchases goods for consumption as decided upon in the plan. The worker enters the labour market and supplies labour according to the intratemporal optimality condition relating consumption and labour supply, taking the real wage as given.⁵

This type of approach has been used in a variety of rational expectations models under imperfect information, including (for example) Lucas (1980) and Rotemberg and Woodford (1999). The latter is particularly relevant as it presents a sticky price model with rational expectations, under the assumption that consumption and pricing decisions are based on information sets including only lagged values of endogenous variables. While they do not formally model the labour

⁴This rules out the case of linear disutility of work in utility ($\gamma = 0$). Given our informational assumptions (to be discussed), this case is problematic because households are unable to forecast their total labour income when labour is demand determined.

⁵This timing assumption is necessary to ensure that the labour supply responds to meet demand. Given our assumptions about price-setting behaviour – explained in Section 2.1.2 – if the household sets either the nominal wage or the amount of labour supplied based solely on date $t - 1$ information, then the real wage is unable to move to the level required to clear the labour market.

market, Rotemberg and Woodford's approach is in fact very similar to ours. They assume that households are producers as well as consumers and include the disutility from producing output in the household utility function. Because prices are set in advance of the shocks hitting the economy in the current period, output is demand determined. This means that households will in general not produce the amount that they expected at the start of the period. In our model, this corresponds to the notion that the 'worker' of the household supplies more or less labour than anticipated at the start of the period.

The household's budget constraint is

$$B_{i,s} - R_{s-1}B_{i,s-1} - W_s h_{i,s} - D_s + P_s c_{i,s} = 0$$

where B is the household's holdings of nominal (one period) bonds, W is the nominal wage, R measures the nominal interest rate paid on bonds, D is a collection of lump-sum transfers (from government and firms) and P is the nominal price level.

Maximisation of utility gives rise to an Euler equation in consumption which can be written in log-linearised form as:

$$\hat{c}_t = E_{t-1} \hat{c}_{t+1} - \sigma^{-1} E_{t-1} (\hat{R}_t - \hat{\Pi}_{t+1}) \quad (1)$$

where we use the notation $\hat{x}_t \equiv \ln(x_t/x)$ for each variable x_t to denote its log-deviation from its steady-state value, x . Our worker-shopper assumption means that the labour supply relationship is given by:⁶

$$\hat{h}_t = \gamma^{-1} \hat{w}_t - \sigma \gamma^{-1} \hat{c}_t \quad (2)$$

2.1.2 Firms

We assume that there are two types of firms – retailers and producers – and we consider each of them in turn.

Retailers are perfectly competitive and operate a production technology that combines the inputs of producers using a Dixit-Stiglitz technology:

$$y_t = z_t \left[\int_0^1 x_{j,t}^{1-\mu} dj \right]^{\frac{1}{1-\mu}}$$

⁶The equation in the text differs from the planned labour supply at the start of the period (given by $\hat{h}_t^p = \gamma^{-1} E_{t-1} \hat{w}_t - \sigma \gamma^{-1} \hat{c}_t$) because the real wage adjusts to clear the labour market.

where x_j is the quantity of output purchased from producer j , z is a productivity shock and $\mu > 0$. We assume that the log-linearised behaviour of the shock z is

$$\hat{z}_t = \rho_z \hat{z}_{t-1} + u_t^z \quad (3)$$

where u^z is an iid Gaussian shock and $|\rho_z| < 1$.

Retailers sell their output to consumers and the government at nominal price P . Denoting the price of output purchased from producer j as P_j , the cost-minimising price index for output is

$$P_t = z_t^{-1} \left[\int_0^1 P_{j,t}^{\frac{\mu-1}{\mu}} dj \right]^{\frac{\mu}{\mu-1}}$$

and the associated demand curve for the output of producer j is:

$$x_{j,t} = z_t^{\frac{1-\mu}{\mu}} \left[\frac{P_{j,t}}{P_t} \right]^{-\frac{1}{\mu}} y_t$$

The set of producers $j \in (0, 1)$ produce differentiated products that form the Dixit-Stiglitz bundle (y , defined above) consumed by households and the government. They produce using a constant returns production function in the single input (labour):

$$x_{j,t} = a_t h_{j,t}$$

where a_t is a stochastic aggregate productivity term (common to all producers) and we assume that log-linearised productivity follows a simple AR(1) process:

$$\hat{a}_t = \rho_a \hat{a}_{t-1} + u_t^a \quad (4)$$

where u^a is an iid Gaussian shock and $|\rho_a| < 1$.

Aggregating the production function across producers, combining with the production function of retailers, and log-linearising gives:⁷

$$\hat{y}_t - \hat{z}_t = \hat{a}_t + \hat{h}_t \quad (5)$$

The real profit of producer j is:

$$\Delta_{jt} = \frac{P_{jt}}{P_t} x_{jt} - w_t h_{j,t} = \left(\frac{P_{jt}}{P_t} - \frac{w_t}{a_t} \right) z_t^{\frac{1-\mu}{\mu}} \left(\frac{P_{j,t}}{P_t} \right)^{-\frac{1}{\mu}} y_t$$

⁷The Dixit-Stiglitz aggregator for output is not equivalent to the simple sum of production functions across firms: there is a wedge between the two measures. This linearisation makes use of the fact that the distortion is second order so can be ignored when considering a linear approximation to the model. See Christiano, Evans and Eichenbaum (2001) for a discussion.

Under a Calvo pricing scheme, the objective function for a producer that is able to reset prices is thus:

$$\max E_{t-1} \sum_{s=t}^{\infty} \Lambda_s (\beta\alpha)^{s-t} \left(\frac{P_{j,t}}{P_s} - \frac{w_s}{a_s} \right) \left(\frac{P_{j,t}}{P_s} \right)^{-\frac{1}{\mu}} z_s^{\frac{1-\mu}{\mu}} y_s$$

where Λ represents the stochastic discount factor of a representative household and $0 \leq \alpha < 1$ is the probability that the producer is *not* allowed to reset its price each period. As in the treatment of households, we assume that the producer's expectations are conditional on the information available up to the end of period $t - 1$.

The log-linearised first-order condition for newly set prices can be combined with the log-linearised expression for the retailer's price index to give:

$$\begin{aligned} \hat{\Pi}_t = & (1 - \alpha)(1 - \beta\alpha) E_{t-1} [\hat{w}_t - \hat{a}_t] + (1 - \alpha) E_{t-1} \hat{\Pi}_t \\ & + \alpha\beta E_{t-1} [\hat{\Pi}_{t+1} + z_{t+1}] - z_t \end{aligned} \quad (6)$$

which is a version of the New Keynesian Phillips curve.⁸

2.1.3 Government and market clearing

The government budget constraint is:

$$B_t^g = R_{t-1} B_{t-1}^g + G_t - P_t \tau_t$$

where B^g is nominal government debt (one period bonds), R is the nominal interest rate, $G (= P \times g)$ is nominal spending and $P \times \tau$ is nominal tax revenue.⁹ In real terms:

$$b_t^g = \frac{R_{t-1}}{\Pi_t} b_{t-1}^g + g_t - \tau_t$$

and we assume that the government issues no debt:

$$B_t^g = b_t^g = 0$$

for all periods t . This means that the government runs a balanced budget each quarter and government spending is financed by (lump sum) tax revenue. The log-linearised expression for

⁸Under the conventional assumption that expectations are based on information up to and including date t we can exploit the fact that $E_t \hat{\Pi}_t = \hat{\Pi}_t$ and equation (6) becomes the standard New Keynesian Phillips curve:

$$\hat{\Pi}_t = \frac{(1 - \alpha)(1 - \beta\alpha)}{\alpha} [\hat{w}_t - \hat{a}_t] + \beta E_t \hat{\Pi}_{t+1} + (\alpha\beta\rho_z - 1) \hat{z}_t$$

where we have also substituted out for expectations of the future shock z_{t+1} using the agents' knowledge of the process driving it.

⁹We consider our model as the 'cashless limit' (Woodford (2003)) of an economy in which households demand fiat money, the issuance of which generates seignoreige for the government. We do so for analytical convenience since the inclusion of money would create an additional choice variable and associated decision rule for households.

government spending is:

$$\hat{g}_t = \rho_g \ln \hat{g}_{t-1} + u_t^g \quad (7)$$

where $0 \leq \rho_g \leq 1$ and u_t^g is an iid Gaussian shock.

Monetary policy is conducted using a Taylor rule with interest rate smoothing and an iid Gaussian shock, which has the log-linearised representation:

$$\hat{R}_t = (1 - \phi_r) \left(\phi_\pi \hat{\Pi}_{t-1} + \phi_y \hat{y}_{t-1} \right) + \phi_r \hat{R}_{t-1} + u_t^R \quad (8)$$

Market clearing dictates that all output is consumed by households or government:

$$\hat{y}_t = (1 - \psi_g) \hat{c}_t + \psi_g \hat{g}_t \quad (9)$$

where ψ_g is a parameter denoting the steady-state share of government expenditure in output.

2.2 Non-rational expectations: ‘Euler equation’ approach

In this section we consider one variant of the model in which agents hold non-rational expectations. We will assume that agents use relatively simple forecasting rules or ‘predictors’ to form their expectations. So in what follows, we will use the term ‘predictors’ as a shorthand for non-rational expectations.

The ‘Euler equation’ approach to modelling non-rational expectations simply replaces the one period ahead rational expectations that appear in the model equations in Section 2.1 with non-rational forecast functions (see for example Bullard and Mitra (2002) and Evans and Honkapohja (2001)). The simplicity of this approach is no doubt an important reason for its popularity. But it is also consistent with a plausible description of boundedly rational behaviour, as discussed by Branch and McGough (2006).

In our model, the Euler equation approach corresponds to simple modifications to just two equations:

$$\hat{c}_t = \hat{E}_{t-1} \hat{c}_{t+1} - \sigma^{-1} \hat{E}_{t-1} \left(\hat{R}_t - \hat{\Pi}_{t+1} \right) \quad (10)$$

$$\begin{aligned} \hat{\Pi}_t = & (1 - \alpha) (1 - \beta \alpha) \hat{E}_{t-1} [\hat{w}_t - \hat{a}_t] + (1 - \alpha) \hat{E}_{t-1} \hat{\Pi}_t \\ & + \alpha \beta \hat{E}_{t-1} [\hat{\Pi}_{t+1} + z_{t+1}] - z_t \end{aligned} \quad (11)$$

Equation (10) is simply equation (1) from the rational expectations model, with the expectation operator E replaced with \hat{E} to denote the fact that the expectation is not rational. Similarly, equation (11) is a version of (6) with non-rational expectations.

We assume that there is heterogeneity in expectations so that for each variable M :

$$\hat{E}_{t-1} M_s = \sum_l q_l \hat{E}_{l,t-1} M_s$$

$$\sum_l q_l = 1$$

where $\hat{E}_{l,t-1}$ is used to denote a particular predictor for M and q_l is the fraction of agents using that predictor. In Section 2.4 below we will specify the types of predictors that agents use. Here as in Section 2.3 below we assume that households (and firms) make their current decisions on the basis of their currently held expectations, without factoring in the possibility that those expectations will evolve as new information arrives. This is the ‘anticipated utility’ assumption that is extensively used in the literature on least squares learning.¹⁰ The anticipated utility approach makes it easier to write down the decision rules of individual households and firms, since they do not depend on the entire sequence of future forecasting rules that may be chosen and is thus very useful in delivering a tractable version of the model.

2.3 *Non-rational expectations: ‘long-horizon’ approach*

In this section, we present an alternative to the Euler equation approach and follow Preston (2005b) to derive agents’ decision rules, taking into account that expectations are non-rational. That is, we solve agents’ optimisation problem conditional on their forecasts of variables relevant to the decision. It follows that ‘long-horizon’ forecasts – the expected path of relevant variables over the lifetime of the agent – matter for decisions. Since different agents may use different predictors and therefore make different decisions, we need to pay attention to the heterogeneity across agents and then aggregate appropriately. Here we focus on how the decision rules of households and firms are affected by the use of non-rational expectations.

¹⁰The anticipated utility approach was introduced by Kreps (1998). Recent work by Cogley and Sargent (2006) indicates that, in some circumstances, behaviour under anticipated utility can outperform that under rational expectations as an approximation to fully optimal behaviour.

2.3.1 Households

As before we start with an individual household among the continuum of unit mass. Household $i \in (0, 1)$ solves:

$$\max \tilde{E}_{i,t-1} \sum_{s=t}^{\infty} \beta^{s-t} \left[\frac{c_{i,s}^{1-\sigma} - 1}{1-\sigma} - \chi \frac{h_{i,s}^{1+\gamma}}{1+\gamma} \right]$$

subject to

$$b_{i,s} - \frac{R_{s-1}}{\Pi_s} b_{i,s-1} - w_s h_{i,s} - d_s + c_{i,s} = 0 \quad (12)$$

where now we use the notation \tilde{E}_i to denote the expectations of household i . The ‘ \sim ’ notation signals that the expectation is not rational and the i subscript makes it clear that the expectation is specific to the individual household.

The Lagrangean for the problem is formed in the same way as in the rational expectations version and gives rise to the same first-order conditions for c_i , h_i and b_i , though with the expectation operator $\tilde{E}_{i,t-1}$ in place of the rational expectations operator E_{t-1} . Appendix A demonstrates that the household’s consumption is given by:

$$\begin{aligned} \hat{c}_{i,t} = & \frac{1-\beta}{1-\psi_g + \frac{\sigma(1-\mu)}{\gamma}} \tilde{E}_{i,t-1} \sum_{s=t}^{\infty} \beta^{s-t} \left(\begin{array}{l} (1-\mu)(1+\gamma^{-1}) \hat{w}_s \\ + \beta^{-1} \tilde{b}_{i,t-1} + (\mu - \psi_g) \hat{d}_s \end{array} \right) \\ & - \frac{\beta}{\sigma} \tilde{E}_{i,t-1} \sum_{s=t}^{\infty} \beta^{s-t} \left(\hat{R}_s - \hat{\Pi}_{s+1} \right) \end{aligned} \quad (13)$$

Equation (13) looks very much like a consumption function: current consumption depends on existing asset holdings plus the expected stream of future net income. So in this case – in contrast to the rational expectations version of the model – long-horizon expectations matter.¹¹ The decision rules above therefore determine the household’s choice variables as a function of the expected path of the variables outside of their control. In the following period, new shocks will have arrived, expectations will be updated and the household will construct a new consumption plan.

The key difference compared with the RE and EE versions is the fact that the household incorporates the (lifetime) budget constraint directly into the consumption decision. Of course,

¹¹The assumption of non-rational expectations is not the only way in which long-horizon expectations may matter. For example, Rotemberg and Woodford (1999) assume that information sets differ across firms so that expectations based on information at dates $t-1$ and $t-2$ are relevant for pricing behaviour at date t . This assumption means that the conventional representation of the New Keynesian Phillips curve cannot be uncovered – so long-horizon expectations remain in their aggregate supply curve – see equation 22 on page 65 of Rotemberg and Woodford (1999).

the Euler equation always describes the optimal relationship between current and future consumption. But to compute the future consumption that it expects to enjoy, the household must factor in forecasts of future net income and real interest rates and ensure that the consumption plan is consistent with the intertemporal budget constraint. In the case of rational expectations, the Euler equation is sufficient to describe the optimal consumption plan because the expectations operator is model consistent: the restrictions of expected consumption from the budget constraint facing the household are taken into account. Evans *et al* (2003) note that if households have access to a *subset* of the information required for rational expectations, then the EE and LH consumption equations are equivalent. This information manifests itself in terms of restrictions on the household's forecasting rules for net income and real interest rates.

2.3.2 Firms

Since, in our model, retailers do not form expectations, their behaviour is unaffected. And the environment faced by producers is essentially the same as that described in Section 2.1.2 though in this case the expectations of producer j is denoted as the (non-rational) expectation $\bar{E}_{j,t-1}$. A producer j given the chance to reset its price maximises:

$$\max \bar{E}_{j,t-1} \sum_{s=t}^{\infty} \Lambda_s (\beta\alpha)^{s-t} \left(\frac{P_{j,t}}{P_s} - \frac{w_s}{a_s} \right) \left(\frac{P_{j,t}}{P_s} \right)^{-\frac{1}{\mu}} z_s^{\frac{1-\mu}{\mu}} y_s$$

which has a first-order condition given by:

$$\bar{E}_{j,t-1} \sum_{s=t}^{\infty} \Lambda_s (\beta\alpha)^{s-t} \left(\frac{\mu-1}{\mu} \frac{P_{j,t}}{\Pi_{t,s}} + \frac{1}{\mu} \frac{w_s}{a_s} \right) \left(\frac{P_{j,t}}{\Pi_{t,s}} \right)^{-\frac{1}{\mu}} z_s^{\frac{1-\mu}{\mu}} y_s = 0$$

where we define the price set by producer j relative to the *previous* period's aggregate price level as:

$$p_{j,t} \equiv \frac{P_{j,t}}{P_{t-1}}$$

and the relative inflation factor

$$\Pi_{t,s} \equiv \frac{P_s}{P_{t-1}} = \Pi_s \times \Pi_{s-1} \times \dots \times \Pi_t \text{ for } s \geq t$$

where we normalise by the aggregate price level from the previous period because this is contained in producers' information set.¹²

¹²Conventional treatments usually define the relative price of a firm j in terms of the current aggregate price level: $p_{j,t} \equiv P_{j,t}/P_t$. There is no loss of generality in following our approach.

Appendix A shows that linearising around the steady state gives the following pricing equation:

$$\hat{p}_{j,t} = (1 - \beta\alpha) \bar{E}_{j,t-1} \sum_{s=t}^{\infty} (\beta\alpha)^{s-t} (\hat{w}_s - \hat{a}_s) + \bar{E}_{j,t-1} \sum_{s=t}^{\infty} (\beta\alpha)^{s-t} \hat{\Pi}_s \quad (14)$$

which makes it clear that if producers have different expectations about future costs and future inflation, then they will set different prices even when free to set them simultaneously.

2.3.3 Aggregation

To continue with the description of the model, we need to aggregate decisions of consumers and firms. We assume that there are a finite set of predictors available to each group of agents. From this point on we will use the index i (j) to denote the decisions of a household (firm) using predictor $i \in \{1, \dots, I\}$ ($j \in \{1, \dots, J\}$). This is appropriate if all agents that use the same predictor make the same decisions, which is the case if there is no dependence of current decisions on past decisions.

Since households have access to financial assets that can be carried between periods, we need to carefully consider the aggregation of household decisions over time. Though the government issues a zero supply of one-period nominal bonds each period, individual consumers can also trade with each other using private bonds. While the total net (private) supply of bonds across the population is zero (by market clearing) an individual household may carry forward a positive or negative bond position from the previous period, reflecting surprise income gains and losses.¹³ Since we know that consumption plans are a linear function of previously accumulated financial assets, we can aggregate across households under reasonable assumptions. For example, deriving aggregated decision rules is straightforward if we assume that the sets of agents in each group (which pins down their view of human wealth) are either fixed or randomly drawn from the population as a whole.¹⁴

However, additional assumptions will in general be required to pin down the consumption paths

¹³Consider a household that starts the period with no financial assets (bonds) and an overly pessimistic view of the stream of future earnings. This household chooses to consume relatively little and receives a positive surprise when income greatly exceeds expenditure during the period and hence ends the period with a higher level of financial assets than they had planned at the start of the period. Similarly, there will be other households that receive a negative surprise about net income and accumulate fewer bonds than they had anticipated.

¹⁴When the proportions are fixed over time, aggregate consumption depends linearly on the total bond holdings of households in the previous period (which sum to zero by market clearing). When the proportions change over time, but are composed of randomly selected households, the law of large numbers implies that the average financial wealth of households in each cohort is zero.

of each *group* of households (and the implied distribution of wealth among these groups). The reason is that each consumption function depends on the level of previously accumulated assets, which themselves evolve over time in response to surprise income gains and losses. This induces a unit root into the consumption and bond holdings of individual households. This is analogous to the behaviour of net foreign assets in small open economy models with infinitely lived consumers (see Barro and Sala-i-Martin (1995), Chapter 3, for a simple exposition). A range of devices have been proposed to deal with this issue in small open economy models and appear to have little effect on the model's dynamic behaviour under rational expectations (see Schmitt-Grohe and Uribe (2003)).

In principle, we could use these devices to ensure that the bond holdings of individual households follow stationary processes. However, we choose to make the assumption that while the fraction of households using each predictor is fixed, the individual households that form each group are drawn randomly from the population as a whole at the start of each period.¹⁵ Since the analysis of the cross-sectional distribution of consumption is not the focus of the analysis, there is no loss of generality in following this approach.

We assume that the mass of households (firms) using predictor i (j) is given by n_i (m_j) where

$$\sum_{i=1}^I n_i = \sum_{j=1}^J m_j = 1$$

This means that the model equations can be written as follows. The average reset price of producers is

$$\begin{aligned} \hat{p}_t^* &= \sum_{j=1}^J m_j \hat{p}_{j,t} \\ &= (1 - \beta\alpha) \sum_{j=1}^J m_j \bar{E}_{j,t-1} \sum_{s=t}^{\infty} (\beta\alpha)^{s-t} (\hat{w}_s - \hat{a}_s) + \sum_{j=1}^J m_{j,t} \bar{E}_{j,t-1} \sum_{s=t}^{\infty} (\beta\alpha)^{s-t} \hat{\Pi}_s \end{aligned}$$

where, for each $j \in \{1, \dots, J\}$, m_j is the fraction of producers setting price \hat{p}_j and inflation is given by:

$$\hat{\Pi}_t = (1 - \alpha) \hat{p}_t^* - \hat{z}_t$$

The linearised resource constraint and monetary policy rule are the same as in the rational

¹⁵An alternative approach would be to view the model as a representative household model in which consumption decisions are based on pooled forecasts of future income flows.

expectations model:

$$\begin{aligned}\hat{y}_t &= (1 - \psi_g) \hat{c}_t + \psi_g \hat{g}_t \\ \hat{R}_t &= (1 - \phi_r) \left(\phi_\pi \hat{\Pi}_{t-1} + \phi_y \hat{y}_{t-1} \right) + \phi_r \hat{R}_{t-1} + u_t^R\end{aligned}$$

Aggregate consumption is:

$$\begin{aligned}\hat{c}_t &= \sum_{i=1}^I n_i \hat{c}_{i,t} \\ &= \frac{1-\beta}{1-\psi_g + \frac{\sigma(1-\mu)}{\gamma}} \sum_{i=1}^I n_i \tilde{E}_{i,t-1} \sum_{s=t}^{\infty} \beta^{s-t} \left[(1-\mu) (1+\gamma^{-1}) \hat{w}_s + (\mu - \psi_g) \hat{d}_s \right] \\ &\quad - \frac{\beta}{\sigma} \sum_{i=1}^I n_i \tilde{E}_{i,t-1} \sum_{s=t}^{\infty} \beta^{s-t} \left(\hat{R}_s - \hat{\Pi}_{s+1} \right)\end{aligned}$$

where we use the fact that bond market clearing requires:

$$\sum_{i=1}^I n_i \tilde{b}_{i,s} = 0$$

for all s .

2.4 Specification of non-rational expectations

In this section we specify a general form for the predictors used by agents in the non-rational expectations versions of the model. We will assume that the proportions of agents using each predictor are exogenous. In the Euler equation approach this amounts to assuming that the fractions q_l , ($l = 1, \dots, L$) are exogenous. For the long-horizon approach we fix the proportions of households and firms – n_i , ($i = 1, \dots, I$) and m_j ($j = 1, \dots, J$) respectively. In Harrison and Taylor (2012), we endogenise the predictor proportions in the long horizon version of the model.

In the rest of this paper, and without loss of generality, we assume that $I = J = L = 2$. For notational convenience we denote $q_1 = q$ so that $q_2 = 1 - q$. Similarly, in the model with long-horizon expectations we set $n_1 = n$, $n_2 = 1 - n$ and $m_1 = m$, $m_2 = 1 - m$. For the greatest level of flexibility, we assume that agents use VARs to form expectations of the variables that they care about. The VARs have the general form:

$$\zeta_t = F \zeta_{t-1} + G \xi_t$$

where $\zeta_t = [\hat{c}_t \ \hat{y}_t \ \hat{\Pi}_t \ \hat{p}_t^* \ \hat{w}_t \ \hat{R}_t \ \hat{d}_t \ \hat{a}_t \ \hat{g}_t \ \hat{z}_t]'$ and $\zeta_t = [u_t^a \ u_t^g \ u_t^r \ u_t^z \ 1]'$. This form permits a wide variety of forecasting models. For example, if we assume that the fifth column of G is equal to a zero vector and specify F so that the largest eigenvalue is less than 1 in magnitude, then

expectations will ultimately converge to the steady-state values of endogenous variables described in Appendix A. Setting both the F and G matrices to zero generates the ‘steady-state predictors’ analysed by Brazier, Harrison, King and Yates (2008).

For the Euler equation approach we note that the expectations terms have the form:

$$\hat{E}_{t-1}\hat{c}_{t+1} = \sum_{l=1}^2 S^c F_l (F_l \zeta_{t-1} + G_l \zeta_t)$$

where we use selector matrices – denoted S – to pick out the forecast for the variable of interest: thus S^x is the matrix that selects the forecast for the variable x .

Applying the VAR expectations under the long horizon approach is a little more involved. For firms, we have:

$$\bar{E}_{j,t}\zeta_{t+s} = F_{f,j}^s (F_{f,j}\zeta_{t-1} + G_{f,j}\zeta_t) \quad \text{for } j = 1, \dots, J$$

and for households:

$$\tilde{E}_{i,t}\zeta_{t+s} = F_{h,i}^s (F_{h,i}\zeta_{t-1} + G_{h,i}\zeta_t)$$

The current period decisions for consumption and prices depend on discounted sums of expected future outturns. We can transform our VAR forecasting model to perform these summations. So for example, for some arbitrary F , G and discount rate $\delta \in (0, 1)$ we have:

$$\begin{aligned} \sum_{s=0}^{\infty} \delta^s \tilde{E}_{t-1}\zeta_{t+s} &= \left(\sum_{s=0}^{\infty} \delta^s F^s \right) (F\zeta_{t-1} + G\zeta_t) \\ &= (I - \delta F)^{-1} (F\zeta_{t-1} + G\zeta_t) \end{aligned}$$

which is valid as long as the eigenvalues of F are all less than δ^{-1} in absolute magnitude.

Two equations in the model contain terms with expectation operators: firms’ pricing equation and households’ consumption equation. Starting with firms, we have:

$$\begin{aligned} \hat{p}_t^* &= \sum_{j=1}^2 m_j \hat{p}_{j,t} \\ &= \sum_{j=1}^2 m_j \left((1 - \beta\alpha) \bar{E}_{j,t-1} \sum_{s=t}^{\infty} (\beta\alpha)^{s-t} (\hat{w}_s - \hat{a}_s) + \bar{E}_{j,t-1} \sum_{s=t}^{\infty} (\beta\alpha)^{s-t} \hat{\Pi}_s \right) \end{aligned}$$

Now we can use the VAR representation of expectations and perform the summations as above

using the VAR coefficient matrices. So the pricing equation can be written as:

$$\hat{p}_t^* = \sum_{j=1}^2 m_j \begin{bmatrix} (1 - \beta\alpha) S^w (I - \beta\alpha F_{f,j})^{-1} (F_{f,j} x_{t-1} + G_{f,j} z_t) \\ - (1 - \beta\alpha) S^a (I - \beta\alpha F_{f,j})^{-1} (F_{f,j} x_{t-1} + G_{f,j} z_t) \\ + S^\pi (I - \beta\alpha F_{f,j})^{-1} (F_{f,j} x_{t-1} + G_{f,j} z_t) \end{bmatrix}$$

Collecting terms, and defining $V_{f,j} \equiv (I - \beta\alpha F_{f,j})^{-1}$ gives:

$$\hat{p}_t^* = \sum_{j=1}^2 m_j [((1 - \beta\alpha) (S^w - S^a) + S^\pi) V_{f,j} (F_{f,j} x_{t-1} + G_{f,j} z_t)]$$

Analogous arguments can be applied to the consumption equation of households giving:

$$\begin{aligned} \hat{c}_t &= \sum_{i=1}^2 n_i \hat{c}_{i,t} \\ &= \sum_{i=1}^2 n_i \left[\left(k_1 S^w + k_2 S^d - \frac{\beta}{\sigma} S^R + \frac{\beta}{\sigma} S^\pi F_{h,i} \right) V_{h,i} (F_{h,i} x_{t-1} + G_{h,i} z_t) \right] \end{aligned}$$

where, $V_{h,i} \equiv (I - \beta F_{h,i})^{-1}$ and

$$k_1 = \frac{(1 - \beta)(1 + \gamma)}{(1 - \psi_g) \frac{\gamma}{1 - \mu} + \sigma}; \quad k_2 = \frac{(1 - \beta)(\mu - \psi_g)}{(1 - \psi_g) + (1 - \mu) \frac{\sigma}{\gamma}}$$

3 Comparing alternative models of non-rational expectations

Our goal in this paper is to assess the extent to which the Euler equation and long-horizon approaches to incorporating non-rational expectations imply differences in the behaviour of the model. To do so, we compare the predictions of three versions of the model: the rational expectations benchmark; the Euler equation version; and the long-horizon version. In this section we report the results of this comparison exercise. We use a common calibration and compare the predictions of each version of the model for the unconditional distribution of the endogenous variables and for the responses of those variables to shocks. We begin in Section 3.1 by detailing the baseline parameter values we use in our experiments. Then we present the comparisons of unconditional distributions and impulse response function in Sections 3.2 and 3.3. Finally, in Section 3.4 we explore the sensitivity of our results to variations in the key parameters of the model.

3.1 Baseline parameter values

Table 1 below documents the choice of the key parameters of the model.

Table 1

α	0.75	β	0.99
γ	0.50	σ_z	0.50
σ	2.26	σ_g	2.14
$\frac{1}{\mu}$	10.0	σ_r	0.97
ψ_g	0.22	σ_a	1.79
ϕ_π	1.50	ρ_g	0.30
ϕ_y	0.125	ρ_z	0.10
ϕ_r	0.25	ρ_a	0.60

The calibration of β is standard in this type of model as is the choice of α (implying a duration of price contracts of one year). We calibrate σ and ψ_g (the coefficient of relative risk aversion and the share of government spending in output) from the estimates of Nelson and Nikolov (2002) using UK data. We also set the variance of the productivity process ($\sigma_a^2 / (1 - \rho_a^2)$) to be the same as their estimates, though we choose a lower persistence (ρ_a). This allows for the possibility that the model may generate endogenous persistence that does not depend on the forcing processes driving the model. Similarly we choose a low value for the autocorrelation of the government spending process (ρ_g) and we set its variance equal to the variance of the productivity process. We assume that the cost-push shocks have a relatively low persistence (ρ_z) and variance (σ_z^2) compared with the other shocks.

The specification of the monetary policy rule implies long-run coefficients on inflation and output (ϕ_π and ϕ_y) in line with Taylor (1993) and a moderate degree of interest rate smoothing $\phi_r = 0.25$. Again, we want to allow the presence of non-rational expectations to have the potential to materially affect the persistence and variability of the variables in the model. The variance of monetary policy shocks is consistent with the midpoint of a range of estimates of Taylor-type rules on UK data. The Frisch elasticity of labour supply (γ^{-1}) is broadly in line with

estimated values in the DSGE literature (see for example Smets and Wouters (2007)). The elasticity of demand in product markets $\frac{1}{\mu}$ is set to imply a mark-up of 10% which is in line with estimates using UK data.

3.2 *Comparing unconditional distributions*

In this section, we compare the unconditional distributions of the model when non-rational expectations are incorporated using the ‘Euler equation’ and ‘long-horizon’ approaches. In Section 3.2.1 we describe how we conduct our comparison. This involves specifying the way that non-rational expectations are incorporated into the Euler equation and long-horizon versions of the model and how the resulting unconditional distributions of endogenous variables are compared. Section 3.2.2 presents the results of this exercise.

3.2.1 *Our approach*

Our approach has two key components. First, we specify the non-rational expectations predictors that will be used in the Euler equation and long-horizon versions and the way we engineer the departure from rational expectations. Second, we specify an approach to measuring the distance between the distributions generated by the different versions of the model.

To make the comparison between the Euler equation and long-horizon versions fair, we assume that the types of predictors used by agents in both versions of the model are the same. Moreover, we assume that the proportions of agents using each predictor are the same in both versions which implies setting $m = n = q$ at all times. We also assume that the two predictors available to agents are ones that perform relatively well. The first predictor is the rational expectations predictor (the F and G matrices that correspond to the rational expectations equilibrium of the model) and the second is non-rational. This setting allows us to define:

$$k = 1 - m = 1 - n = 1 - q$$

When $k = 0$ all agents (in both the Euler equation and long-horizon versions) use the ‘rational expectations predictor’ and both versions of the model behave identically to the rational expectations benchmark. Varying k therefore allows us to control the departure from rational expectations since k represents the proportion of agents with ‘non-rational’ expectations.



As noted above, we assume that the ‘non-rational’ predictor performs reasonably well. Specifically, we assume that the predictor is defined as a diagonal F matrix so that agents are constrained to forecast each variable using a univariate AR(1) model.¹⁶ However, we assume that the coefficients of the AR(1) models are optimal in the sense that they provide the best AR(1) forecasts conditional on the structure of the economy. Formally, we find the AR(1) coefficients that represent the fixed point in the mapping between the perceived law of motion (of agents using the AR(1) predictor) and the actual law of motion. To do so, we guess values for the AR(1) coefficients, solve for the VAR representation of the model and compute the asymptotic autocorrelation coefficients for each variable. We then refine our guess of the parameter values until it coincides with the asymptotic autocorrelations.¹⁷

There are two notable implications of this approach. First, the actual predictors used by agents in the Euler equation and long-horizon versions will differ, since the AR(1) coefficients are chosen conditional on the assumed structure of each model (including the way in which expectations affect decisions). This approach is designed to make sure that the predictors used by agents perform relatively well in the context that they are used. An alternative approach is to use the same non-rational predictors in each version of the model. In our view, it is difficult to quantify deviations of a particular non-rational predictor from rational expectations independently of the model in question. However, as a cross-check on our results, we have repeated our experiments in the case where exactly the same F matrices are used in each model – and these results are reported as part of our sensitivity analysis. The second implication is that the AR(1) coefficients depend on k . As k approaches unity, virtually no agents use the rational expectations predictor and the AR(1) coefficients of the forecasting models represent restricted perceptions equilibria of each version of the model.

The second component of our approach is to specify a way of measuring the differences in the behaviour of the two non-rational expectations models. We choose to gauge the extent of these deviations using the Kullback-Leibler (KL) distance which provides a measure of the difference between two multivariate probability distributions. This type of approach (also known as relative entropy) has been used by Robertson, Tallman and Whiteman (2002) to produce conditional

¹⁶The G matrix is zero, reflecting the assumption that agents do not observe current dated shocks.

¹⁷Such a fixed point may not exist, and if it does may not be unique (although the convergence of our algorithm can be taken to establish a form of learnability).

forecasts from VAR models and to measure the distortion to the forecast distribution implied by the imposition of the conditioning assumptions. Cogley, Morozov and Sargent (2005) use similar methods to measure the extent to which the forecast distributions published in Bank of England *Inflation Reports* differ from those generated by a time-varying Bayesian VAR model.¹⁸

To introduce the KL distance, consider two probability density functions defined for a multivariate random variable $x \in X \subseteq \mathbb{R}^{n_x}$ and denoted $f(x)$ and $g(x)$.¹⁹ Then the KL distance is defined as:

$$\mathbb{K}_f \equiv \int_{x \in X} f(x) \ln \left(\frac{f(x)}{g(x)} \right) dx$$

which is also known as the relative entropy of density f with respect to density g . Of course, we can also examine the relative entropy of density g with respect to f which is defined as:

$$\mathbb{K}_g \equiv \int_{x \in X} g(x) \ln \left(\frac{g(x)}{f(x)} \right) dx$$

and it is clear that in general $\mathbb{K}_f \neq \mathbb{K}_g$.²⁰ The fact that these measures are not identical means that the KL distance is not a true metric,²¹ so in our applications we ‘standardise’ the measure by taking an average of the two:

$$\bar{\mathbb{K}} \equiv \frac{\mathbb{K}_f + \mathbb{K}_g}{2}$$

Constructing the KL measure for our models is relatively straightforward. The discussion in Section 2.4 indicates that, because agents’ expectations are specified as VAR models, the actual law of motion will also take a VAR form. The coefficients of the VAR will depend on the way that expectations are modelled (ie the ‘Euler equation’ or ‘long-horizon’ approach) and the proportion of agents, k , forming expectations using the ‘RPE predictor’. The VAR form of the actual law of motion of the economy is denoted

$$X_t = \Gamma_v(k) X_{t-1} + \Psi_v(k) \varepsilon_t$$

where we use $v = EE, LH$ to denote the coefficient matrices corresponding to the Euler equation and long-horizon approaches respectively, X collects the endogenous variables of the

¹⁸Of course, this is not the only way of measuring the differences between the models under alternative expectations schemes. We are grateful to an anonymous referee for suggesting a comparison in terms of the welfare of economic agents under alternative assumptions about expectations.

¹⁹Fairly weak assumptions (satisfied in all cases we consider) are needed to ensure that the Kullback-Leibler distance is well defined. In particular we require $f(x), g(x) > 0$ for $\forall x \in X$.

²⁰The only exception is when the densities are identical ($f(x) = g(x), \forall x \in X$) when $\mathbb{K}_f = \mathbb{K}_g = 0$.

²¹See Robertson *et al* (2002) for a discussion of the interpretations of the two versions in estimation exercises.

model (possibly including lags of those variables) and ε denotes the vector of Gaussian shocks (u_t^a, u_t^s, u_t^R and u_t^z) standardised to have unit variance.

The unconditional distribution of X is then given by

$$X \sim N(0, \Sigma_v(k))$$

where the covariance matrix satisfies the discrete Lyapunov equation

$$\Sigma_v(k) = \Gamma_v(k) \Sigma_v(k) \Gamma_v(k)' + \Psi_v(k) \Psi_v(k)'$$

Because X has a multivariate normal distribution, we can substitute the formula for the probability density functions of X directly into the formulae for the Kullback-Leibler distance to obtain:

$$\begin{aligned} \mathbb{K}_{EE}(k) &= [(2\pi)^{n_x} |\Sigma_{EE}(k)|]^{-1/2} \int_{x \in X} \exp \left[-\frac{1}{2} x' \Sigma_{EE}^{-1}(k) x \right] \frac{x' \Sigma_{EE}^{-1}(k) x}{x' \Sigma_{LH}^{-1}(k) x} dx \\ &= \frac{1}{2} \left(\ln \left(\frac{|\Sigma_{LH}(k)|}{|\Sigma_{EE}(k)|} \right) + \text{tr}(\Sigma_{LH}^{-1}(k) \Sigma_{EE}(k)) - n_x \right) \end{aligned}$$

where n_x is the number of elements in X . $\mathbb{K}_{LH}(k)$ is constructed analogously.

3.2.2 Results

To assess the extent to which the alternative methods of incorporating non-rational expectations differ as we vary k (the fraction of agents using non-rational expectations), we simply plot the KL distance $\bar{\mathbb{K}}$ against the proportion k . Before inspecting these charts it is worth noting two points.

First, note that setting $k = 0$ delivers Γ and Ψ matrices that coincide with the rational expectations equilibrium in both the ‘Euler equation’ and ‘long-horizon’ cases. So when $k = 0$ both approaches to incorporating non-rational expectations predict identical distributions for the endogenous variables so that $\bar{\mathbb{K}} = 0$.

Second, note that the number of shocks hitting the model is less than the number of endogenous variables. This means that the covariance matrix of the vector of endogenous variables is of reduced rank and in this case $\bar{\mathbb{K}}$ is undefined. To avoid this problem, we need to choose a subset of endogenous variables for which to evaluate $\bar{\mathbb{K}}$. Since there are nine endogenous variables and four shocks, there are a large number of possible choices that can be made. Although the values

computed for $\bar{\mathbb{K}}$ depend on this choice, they do so in an intuitive way; and the qualitative pattern of the results is not sensitive to the choice of variables. To illustrate these points we include plots of the KL distance between the model variants for two representative subsets of variables.

Chart 1 shows values for $\bar{\mathbb{K}}$ as k is varied, where $\bar{\mathbb{K}}$ is computed as the KL distance between the joint distributions of $[\pi, w, R, y]$ in the two versions of the model (these variables are selected on account of their economic importance). As the proportion of agents using non-rational expectations increases, the gap between the two variants of the model, as measured by $\bar{\mathbb{K}}$, tends to increase. Chart 2 shows the KL distance for a second subset of variables, $[p, w, R, c]$. This group of variables includes the two key decision variables for private agents in the model - households' consumption and firms' prices.²² Again the KL distance between the Euler equation and long-horizon variants of the model increases as the proportion of agents using non-rational expectations increases. However it is evident that the gap between the joint distributions of these variables in the two versions of the model is larger than in Chart 1. The reason is that expectations enter the model through agents' decisions, and different approaches to incorporating expectations affect model properties via changes in the distribution of agents' decisions. So the difference between the model variants is most clearly seen in a measure of the gap between the joint distributions of a set of variables that includes the key choice variables. Of course the distributions of π and y (and other variables) are also affected by the expectational assumptions; but π and y are composites of the distribution of choice variables and of exogenous processes (z and g respectively). So the behaviour of these whole-economy variables is less affected by the change in the way expectations are incorporated.

3.3 Comparing impulse responses

Impulse responses provide an alternative perspective on the extent to which the 'Euler equation' and 'long-horizon' approaches to incorporating non-rational expectations affect the properties of our model. We present two panels of impulse responses, each showing the response of three key model variables to the four shocks for a different value of k . Each chart shows the impulse response under rational expectations, as a benchmark. The other two lines in each chart show the

²²A feature of the Euler equation approach is that the behavioural equations are not consistent with an underlying optimisation problem. In particular the Euler equation variant of the model that we presented in Section 2.3 does not specify a pricing equation consistent with the expectational assumption embodied in the Phillips curve (11). So to recover a distribution for prices we compute the implied aggregate reset price as $\hat{p}_t^* = \frac{\hat{\pi}_t - \hat{z}_t}{(1-\alpha)}$.

Chart 1: KL distance for $[\pi, w, r, y]$

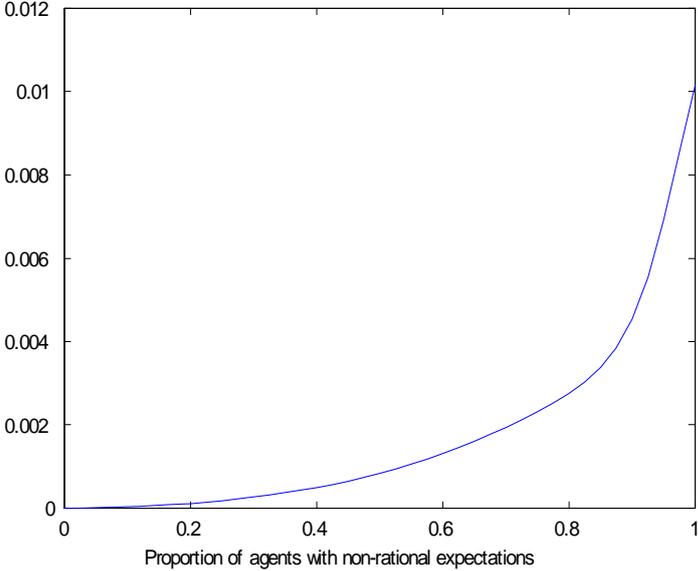
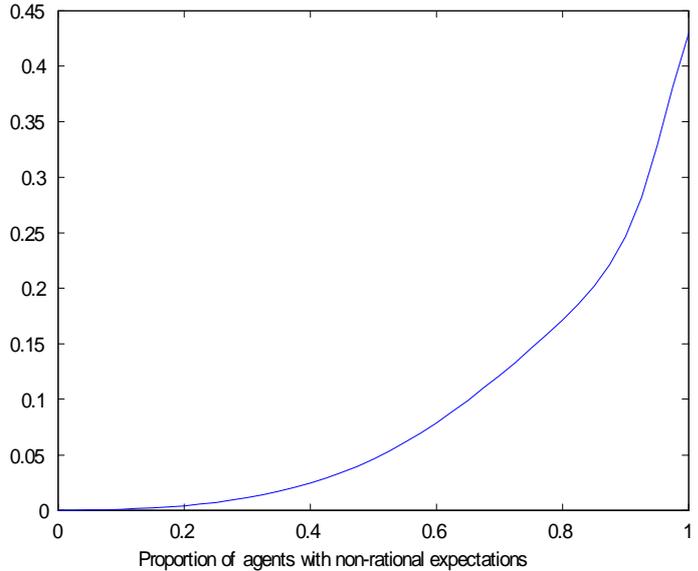
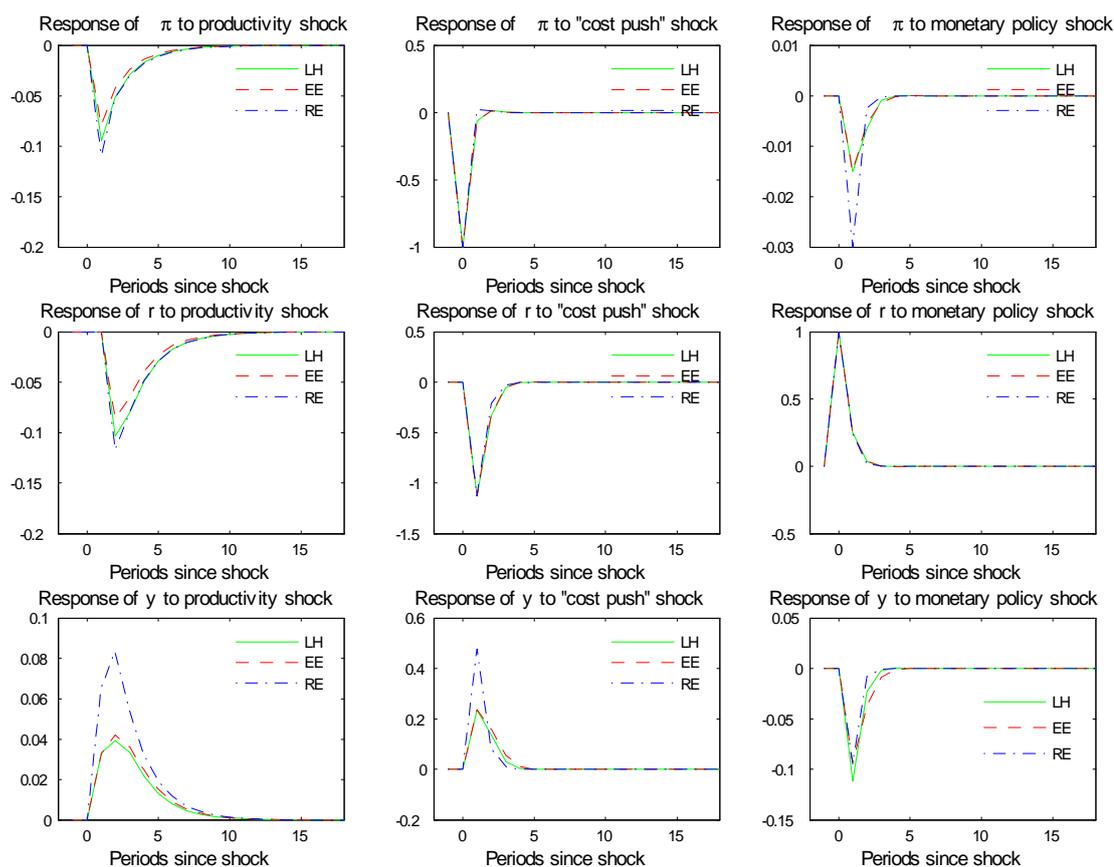


Chart 2: KL distance for $[p, w, r, c]$



responses of the Euler equation and long-horizon variants of the model. In Chart 3 the proportion of agents using non-rational expectations is fixed at half (that is, $k = 0.5$),²³ and in Chart 4 all agents are assumed to have non-rational expectations ($k = 1$).

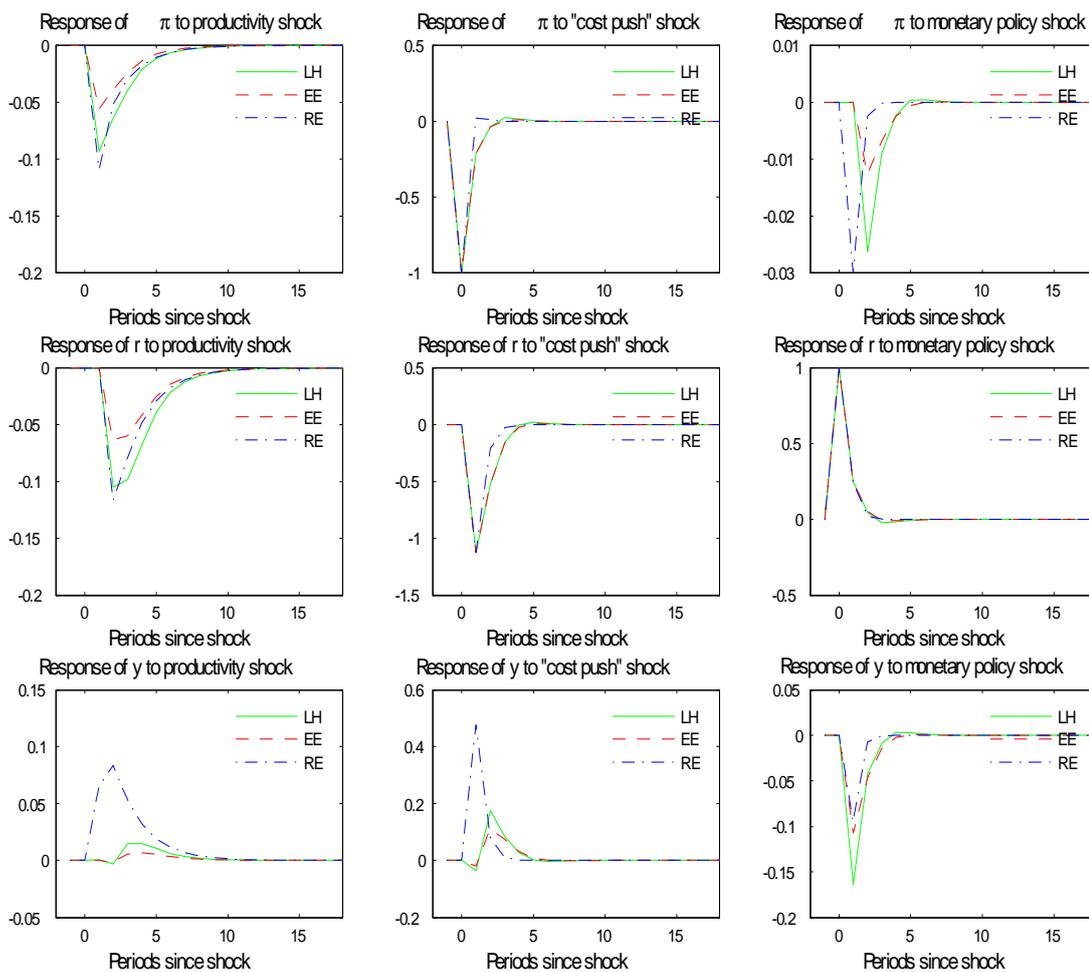
Chart 3: Impulse responses with $k = 0.5$ for non-rational Euler equation (EE) and non-rational long-horizon (LH) versions of the model, compared to rational expectations (RE)



When half of the agents in the model are assumed to form expectations non-rationally the dynamics of the two variants of the model, as represented by the impulse responses in Chart 3, are similar. However noticeable differences arise as the proportion of agents using non-rational expectations increases. In the limit where $k = 1$ some responses are markedly different, as shown in Chart 4. In particular the response of inflation to the productivity shock is markedly larger in the long-horizon variant of the model than in the Euler equation variant. This is due to the

²³Impulse responses for values of k less than 0.5 are very similar to those shown in Chart 3.

Chart 4: Impulse responses with $k = 1$ for non-rational Euler equation (EE) and non-rational long-horizon (LH) versions of the model, compared to rational expectations (RE)



relatively high persistence of the productivity process, which brings into sharper relief the different implementations of expectations.²⁴

Indeed, (relative to the EE responses) the LH responses are often closer to the RE responses. This is unsurprising given the fact that the long-horizon expectations in the pricing and consumption equations correctly forecast the ‘present value’ of the relevant shocks (eg productivity, ‘cost-push’) and so, in the LH version, these are incorporated into pricing consumption decisions. For example, a noticeable result from 4 is that the responses of interest rates and inflation to the productivity shock under LH expectations are more similar to RE than the EE responses. The similarity in the interest rate response is driven by the similarity in the inflation responses (since the Taylor rule is a simple feedback rule from inflation to interest rates). And the inflation response in the LH case is similar to the RE version because firms perfectly foresee the contribution of productivity to the expected stream of future marginal costs.

Another noteworthy feature of the impulse responses is the limited output gap effect of supply shocks (ie productivity and ‘cost-push’ shocks) under either variant of the model with non-rational expectations, compared to the rational expectations case. This is because moving to non-rational expectations weakens the transmission from the supply side of the model to households’ demand. Transmission happens primarily through two channels: wages and nominal interest rates. While the labour market response to the shocks is similar at first under rational and non-rational expectations, the anticipated path of monetary policy differs. Rational agents foresee the full extent of the policy response (which is small at first but grows) and adjust their consumption accordingly. Non-rational agents expect the policy response to decay as an AR process, and do not factor in the full policy loosening which eventually occurs when making their consumption decision: hence the attenuated response of output.

3.4 Sensitivity analysis

In this section we investigate the sensitivity of our results to alternative assumptions, along two distinct dimensions. First we investigate how our measure of the difference between the Euler equation and long-horizon variants of the model, $\bar{\mathbb{K}}$, varies when we change key model

²⁴We will return to this point in our sensitivity analysis, where we find that increasing the persistence of the cost-push shock also magnifies the difference between the model variants.

parameters. Then we hold the parameters fixed at their baseline values and consider a range of alternative non-rational predictors, to show that our results are not dependent on the way we motivated and set up non-rational expectations (explained in Section 3.2.1)

3.4.1 Sensitivity to alternative parameter values

Table 2 reports values for $\bar{\mathbb{K}}$ under different parameterisations. The first two columns show results for $[\pi, w, r, y]$, while the third and fourth columns report results for $[p, w, r, c]$. To limit the size of the table we report $\bar{\mathbb{K}}$ only for $k = 0.25$ and $k = 0.75$.²⁵ Each row corresponds to a particular parameter vector. The baseline parameters, results for which are reported in the first row, were detailed in Table 1 above. In the subsequent rows we vary the values of key parameters, one at a time, while leaving the remainder at their baseline values (note though that in the case of ρ_z and ϕ_r we adjust the innovation variance so as to offset the effect of the change in persistence on the unconditional variance of z and r). On the left hand side of the table we report which parameter has been altered from its baseline value, and the value to which it is set.

²⁵As k approaches 1 our fixed point algorithm does not converge under some of these parameter combinations.

Table 2

	Variables: $[\pi, w, r, y]$		Variables: $[p, w, r, c]$	
	$k = 0.25$	$k = 0.75$	$k = 0.25$	$k = 0.75$
Baseline	0.0002	0.0023	0.0076	0.1456
$\rho_z = 0.33$	0.0004	0.0031	0.0077	0.0825
$\rho_z = 0.66$	0.0118	0.9705	0.0372	2.3993
$\sigma = 5$	0.0002	0.0023	0.0080	0.1514
$\sigma = 1$	0.0003	0.0028	0.0071	0.1393
$\gamma = 0.25$	0.0002	0.0014	0.0092	0.1513
$\gamma = 2$	0.0008	0.0125	0.0117	0.1634
$\alpha = 0.65$	0.0008	0.0075	0.0101	0.1154
$\alpha = 0.85$	0	0.0005	0.0091	0.1946
$\phi_\pi = 1.01$	0.0002	0.0028	0.0060	0.1398
$\phi_\pi = 5$	0.0002	0.0138	0.0156	0.2077
$\phi_r = 0.5$	0.0017	0.0070	0.0095	0.1385
$\phi_r = 0.75$	0.0091	0.0254	0.0208	0.2865
$\phi_y = 0.5$	0.0002	0.0026	0.0061	0.1325

We draw attention to two key features of these results. First, when a larger proportion of agents use non-rational expectations the measured gap between the model variants is larger in all cases presented. That said, we cannot make any general statements about the relationship between $\bar{\mathbb{K}}$ and k . Although $\bar{\mathbb{K}}$ appears to be increasing in k , it does not always do so monotonically.

The second notable feature of the results presented in Table 2 is the high sensitivity of $\bar{\mathbb{K}}$ to increases in ρ_z and ϕ_r . These parameters strongly influence the degree of persistence in the processes for the endogenous variables; and therefore the degree of persistence embodied in agents' expectations. Intuitively, it is when agents believe that economic variables are persistent that the difference between long-horizon and Euler equation decision rules under non-rational expectations is likely to be largest.

3.4.2 Sensitivity to alternative predictors

So far in our analysis the forecasting rules used by the agents in our model have been rather sophisticated. That is to say, they perform relatively well because they are chosen to be optimal within a set of possible forecasting rules. In part this approach is a response to a challenge posed by the comparison exercise. Agents in the Euler equation variant of the model are required to forecast a different set of variables to those in the long-horizon variant of the model.

The challenge lies in how to set up the forecasting rules in the two variants of the model so as ensure a ‘fair’ comparison. As we have explained, our approach has been to determine the non-rational predictor as the optimal forecasting rule within a class of simple rules, given the structure of the model. A consequence is that the non-rational expectations which enter the two variants of the model may differ. For example, for a given value of k the inflation expectations rule used in the long-horizon variant of the model will almost certainly differ from that in use in the Euler equation variant.

In this section we check that our qualitative results are robust to alternative ways of specifying agents’ expectations, in which we implement exactly the same expectations in the two variants of the model. That is to say, we impose the same F matrices across the two model variants.²⁶ In addition, relaxing the requirement that the non-rational predictor is chosen ‘optimally’ allows us to consider cases that are much further from rational expectations. We show that in some instances much larger measured differences between the model variants can arise.

We consider three illustrative non-rational predictors, again restricting our attention to univariate AR(1) forecasting rules, so that the F matrices are diagonal. The first projects each variable using its first-order autocorrelation under rational expectations. The second has agents believing all variables to be persistent, with a serial correlation of 0.8. And the third illustrative case is one in which all variables are expected to be close to their steady-state values at all times, having a serial correlation of 0.01. In practice these simple predictors may perform poorly as forecasting rules for some or all values of k . However they allow us to explore how the Euler equation and long-horizon variants of the model differ when expectations are far from rational. Table 3 shows

²⁶Here k can more straightforwardly be interpreted as capturing the extent of deviation from rational expectations.

results for our baseline set of variables, $[\pi, w, r, y]$, for $k = 0.25$ and $k = 0.75$, using the same parameter combinations as in our previous sensitivity analysis.

Table 3

	Expectations assumption:					
	AR approx to RE		Persistent		Steady state	
	k = 0.25	k = 0.75	k = 0.25	k = 0.75	k = 0.25	k = 0.75
Baseline	0.0002	0.0018	0.2142	2.4039	0	0
$\rho_z = 0.33$	0.0002	0.0029	0.1623	2.3098	0	0
$\rho_z = 0.66$	0.0035	0.0148	0.0996	1.9574	0	0
$\sigma = 5$	0.0002	0.0015	0.1656	1.9927	0	0
$\sigma = 1$	0.0003	0.0030	0.3771	3.7566	0	0
$\gamma = 0.25$	0.0002	0.0027	0.4085	3.0163	0	0
$\gamma = 2$	0.0005	0.0032	0.0864	3.3557	0	0
$\alpha = 0.65$	0.0007	0.0052	0.2296	5.8592	0	0
$\alpha = 0.85$	0	0.0007	0.1807	1.6142	0	0
$\phi_\pi = 1.01$	0.0002	0.0019	0.2139	1.9001	0	0
$\phi_\pi = 5$	0.0002	0.0017	0.5444	NA	0	0
$\phi_r = 0.5$	0.0025	0.0277	0.2040	2.0041	0	0
$\phi_r = 0.75$	0.0180	0.2094	0.1463	1.4222	0	0
$\phi_y = 0.5$	0.0002	0.0018	0.2445	2.7703	0	0

NA indicates non-convergence of the fixed point algorithm

The results in the first two columns of Table 3 are similar to those in the first two columns of Table 2. This shows that the way we select the non-rational predictors in Section 3.2, in particular the fact that they vary with k and with the implementation method, is not crucial to our results. In fact the non-rational predictors underlying these sets of results are similar, in that both are AR(1) predictors chosen to have reasonable properties. But when the non-rational predictor is chosen in a more arbitrary way the two model variants may differ more markedly. For example, in the case where agents are assumed to believe that all variables share a high degree of

persistence, the difference between the two variants of the model is considerably larger. However in the case where all variables are expected to return to close to their steady-state values, there is a negligible difference between the two model variants: terms in expectations are always close to zero, and play virtually no role in determining decisions, so that the way non-rational expectations are incorporated ceases to matter. Although this is something of a special case, it reminds us that we cannot claim an unconditional relationship between the distance from rational expectations and the properties of the long-horizon and Euler equation implementations of non-rational expectations.

Nevertheless, while deriving a general result for all classes for forward-looking models is well beyond the scope of this paper, our results do suggest that the discrepancy between EE and LH approaches generally increases as the deviation from rational expectations (as measured by k) increases. Intuitively, this result is weaker (though still present) in the case where agents believe that macroeconomic variables are less persistent. The reason is straightforward: the LH approach means that agents respond to the ‘present value’ of future variables, which is very similar to the one step ahead projection when there is little persistence.

4 Conclusions

We compare variants of a small calibrated macroeconomic model under two approaches to the implementation of non-rational expectations. One approach takes the equations of the model under rational expectations and replaces the expectations terms with alternative processes for expectations; and the second approach solves the decision rules of households and firms conditional on their expectations for future events that are outside of their control so that spending and price-setting decisions depend on long-horizon expectations.

We show that the approach taken to characterising behaviour under non-rational expectations can have a material impact on model properties as captured by impulse responses and relative entropy measures of the distance between the probability distributions for the endogenous variables. In general, the more expectations deviate from rational expectations, the greater is the difference in the model variants. When expectations are non-rational but relatively close to model-consistency, the difference between the model variants is likely to be small.



From the perspective of a researcher contemplating how to implement non-rational expectations in a model, the extent to which expectations are thought to deviate from rational expectations should be an important consideration in deciding which approach to take.



Appendix A: Derivation of the model

A.1 The rational expectations case

A.1.1 Households

As noted in the main text, household i solves:

$$\max E_{t-1} \sum_{s=t}^{\infty} \beta^{s-t} \left[\frac{c_{i,s}^{1-\sigma} - 1}{1-\sigma} - \chi \frac{h_{i,s}^{1+\gamma}}{1+\gamma} \right]$$

subject to

$$b_{i,s} - \frac{R_{s-1}}{\Pi_s} b_{i,s-1} - w_s h_{i,s} - d_s + c_{i,s} = 0$$

The Lagrangean for the problem is:

$$\max E_{t-1} \sum_{s=t}^{\infty} \beta^{s-t} \left[\begin{array}{c} (1-\sigma)^{-1} (c_{i,s}^{1-\sigma} - 1) - \frac{\chi}{1+\gamma} h_{i,s}^{1+\gamma} \\ -\lambda_{i,s} \left(b_{i,s} - \frac{R_{s-1}}{\Pi_s} b_{i,s-1} - w_s h_{i,s} - d_s + c_{i,s} \right) \end{array} \right]$$

and the first-order conditions for c_i , h_i and b_i are:

$$E_{t-1} [c_{i,s}^{-\sigma} - \lambda_{i,s}] = 0 \quad (\text{A-1})$$

$$E_{t-1} [-\chi h_{i,s}^{\gamma} + \lambda_{i,s} w_s] = 0 \quad (\text{A-2})$$

$$E_{t-1} \left[-\lambda_{i,s} + \frac{\beta R_s}{\Pi_{s+1}} \lambda_{i,s+1} \right] = 0 \quad (\text{A-3})$$

A.1.2 Firms

As noted in the text, the first-order condition for a producer resetting its price at date t is:

$$E_{t-1} \sum_{s=t}^{\infty} \Lambda_s (\beta \alpha)^{s-t} \left(\frac{\mu-1}{\mu} \frac{1}{P_s} + \frac{1}{\mu} \frac{w_s}{P_{j,t} a_s} \right) \left(\frac{P_{j,t}}{P_s} \right)^{-\frac{1}{\mu}} z_s^{\frac{1-\mu}{\mu}} y_s = 0$$

or

$$E_{t-1} \sum_{s=t}^{\infty} \Lambda_s (\beta \alpha)^{s-t} \left(\frac{\mu-1}{\mu} \frac{p_{j,t}}{\Pi_{t,s}} + \frac{1}{\mu} \frac{w_s}{a_s} \right) \left(\frac{p_{j,t}}{\Pi_{t,s}} \right)^{-\frac{1}{\mu}} z_s^{\frac{1-\mu}{\mu}} y_s = 0 \quad (\text{A-4})$$

if we define the price set by firm j relative to the previous period's aggregate price level as:

$$p_{j,t} \equiv \frac{P_{j,t}}{P_{t-1}}$$



and the relative inflation factor

$$\Pi_{t,s} \equiv \frac{P_s}{P_{t-1}} = \Pi_s \times \Pi_{s-1} \times \dots \times \Pi_t \text{ for } s \geq t$$

where we normalise by the aggregate price level from the *previous* period because this is contained in firms' information set.

Since all firms are identical in terms of their information and production constraints, all firms that are able to change prices at date t will choose the same price, which we denote as p_t^* . Thus

$$E_{t-1} \sum_{s=t}^{\infty} \Lambda_s (\beta\alpha)^{s-t} \left(\frac{\mu-1}{\mu} \frac{p_t^*}{\Pi_{t,s}} + \frac{1}{\mu} \frac{w_s}{a_s} \right) \left(\frac{p_t^*}{\Pi_{t,s}} \right)^{-\frac{1}{\mu}} z_s^{\frac{1-\mu}{\mu}} y_s = 0$$

The retailer's price is:

$$\begin{aligned} P_t &= z_t^{-1} \left[\int_0^1 P_{j,t}^{\frac{\mu-1}{\mu}} dj \right]^{\frac{\mu}{\mu-1}} \\ &= z_t^{-1} \left[\sum_{k=0}^{\infty} (1-\alpha) \alpha^k (P_{t-k}^*)^{\frac{\mu-1}{\mu}} \right]^{\frac{\mu}{\mu-1}} \end{aligned}$$

where the equality follows from grouping the firms into cohorts according to the date at which they last reset their price and noting that the mass of firms that have not reset their price since date $t-k$ is $(1-\alpha)\alpha^k$. This means that the aggregate price level can be written as

$$P_t = z_t^{-1} \left[\alpha P_{t-1}^{\frac{\mu-1}{\mu}} + (1-\alpha) (P_t^*)^{\frac{\mu-1}{\mu}} \right]^{\frac{\mu}{\mu-1}}$$

so that

$$1 = \alpha \left(\frac{1}{z_t \Pi_t} \right)^{\frac{\mu-1}{\mu}} + (1-\alpha) \left(\frac{p_t^*}{z_t \Pi_t} \right)^{\frac{\mu-1}{\mu}}$$

A.1.3 Government and market clearing

The government budget constraint is:

$$B_t^g = R_{t-1} B_{t-1}^g + G_t - P_t \tau_t$$

where B^g is nominal government debt (one period bonds), R is the nominal interest rate, G is nominal spending and $P \times \tau$ is nominal tax revenue. In real terms:

$$b_t^g = \frac{R_{t-1}}{\Pi_t} b_{t-1}^g + g_t - \tau_t$$

and we assume that the government issues no debt:

$$B_t^g = b_t^g = 0$$



for all periods t . This means that the government runs a balanced budget each quarter and government spending is financed by tax revenue. Government spending follows an exogenous process:

$$\ln g_t = \rho_g \ln g_{t-1} + (1 - \rho_g) \ln g + u_t^g$$

where $0 \leq \rho_g \leq 1$ and u_t^g is iid. We use g to denote the steady-state level of government spending.

Monetary policy is conducted using a Taylor rule with interest rate smoothing and an iid shock:

$$\frac{R_t}{R} = \left[\left(\frac{\Pi_{t-1}}{\Pi} \right)^{\phi_\pi} \left(\frac{y_{t-1}}{y} \right)^{\phi_y} \right]^{1-\phi_r} \left(\frac{R_t}{R} \right)^{\phi_r} \exp[u_t^R]$$

where again variables without subscripts are steady-state values.

Market clearing dictates that all output is consumed by households or government:²⁷

$$y_t = c_t + g_t$$

A corollary of this market clearing condition is that the market for nominal one period bonds also clears. This requires that

$$b_t^{agg} = b_t + b_t^g = 0$$

where b_t^{agg} denotes the real-valued supply of all (private and publicly issued) nominal bonds. Given that government debt issuance is zero, we require that the net supply of private bonds is zero: $b_t = 0$. Because all households are identical, this means that no consumer issues or holds debt in equilibrium. Another implication of this assumption is that all households choose the same level of consumption:

$$c_{i,t} = c_t$$

As noted in Section 2.1.1, the ‘worker’ of the household supplies labour according to the intratemporal labour supply condition based on the market real wage:

$$\chi h_{i,t}^\gamma = c_{i,t}^{-\sigma} w_t \tag{A-5}$$

²⁷This holds under the conventional assumption that households and government consume the same basket of goods, which they purchase at the same price.

Finally, the total transfers to consumers consist of dividends net of lump-sum taxes:

$$d_t = \Delta_t - \tau_t$$

in equilibrium this is given by:²⁸

$$d_t = y_t - w_t h_t - g_t$$

or

$$d_t = c_t - w_t h_t$$

A.1.4 Steady state

We now consider a steady state around which the model equations will be linearised. We assume that steady-state inflation (ie the inflation target) is $\Pi = 1$ which means that the Euler equation gives

$$R = \beta^{-1}$$

Since steady-state productivities are $a = z = 1$, this immediately implies:

$$y = h$$

and from market clearing and the specification of steady state government spending, we have:

$$y = c + g = c + \psi_g y$$

so that:

$$c = (1 - \psi_g) y$$

where ψ_g is the (exogenous) steady-state share of government procurement in output.

From the pricing equation in steady state, we see that:

$$w = 1 - \mu$$

Using the results above in the labour supply function implies

$$\chi h^{\gamma+\sigma} = (1 - \psi_g)^{-\sigma} (1 - \mu)$$

²⁸This equality follows from the fact that firm's output x is remunerated at a price equal to the aggregate price level adjusted by the factor z^{-1} as shown in the main text. Since (to a first-order approximation) $y = z \cdot x$ the revenue from selling x is equivalent to the value of retail output.

which means that if we choose the weight on disutility from work to be

$$\chi = (1 - \psi_g)^{-\sigma} (1 - \mu)$$

then the steady-state solution satisfies

$$y = h = 1$$

These observations allow us to express steady-state dividends as:

$$d = c - wh = (1 - \psi_g - (1 - \mu))y = \mu - \psi_g$$

A.1.5 Log-linearised model under rational expectations

We now log-linearise the model equations around the steady state analysed above. We denote $\hat{k}_t \equiv \ln(k_t/k)$ as the log deviation of k_t from its steady-state level k . Linearising the first-order condition for consumption (A-1) and labour supply (A-5) (using the linearised first-order condition for bond holdings (A-3)) gives

$$\hat{c}_t = E_{t-1}\hat{c}_{t+1} - \sigma^{-1}E_{t-1}(\hat{R}_t - \hat{\Pi}_{t+1}) \quad (\text{A-6})$$

$$\hat{h}_t = \gamma^{-1}\hat{w}_t - \sigma\gamma^{-1}\hat{c}_t \quad (\text{A-7})$$

The linearised resource constraint is

$$\hat{y}_t = (1 - \psi_g)\hat{c}_t + \psi_g\hat{g}_t \quad (\text{A-8})$$

and the process for government spending satisfies

$$\hat{g}_t = \rho_g\hat{g}_{t-1} + u_t^g \quad (\text{A-9})$$

The linearised monetary policy rule is

$$\hat{R}_t = (1 - \phi_r)\left(\phi_\pi\hat{\Pi}_{t-1} + \phi_y\hat{y}_{t-1}\right) + \phi_r\hat{R}_{t-1} + u_t^R \quad (\text{A-10})$$

The production function implies that, to a first-order approximation:

$$\hat{y}_t - \hat{z}_t = \hat{a}_t + \hat{h}_t \quad (\text{A-11})$$

where the productivity shocks follow the processes

$$\hat{a}_t = \rho_a\hat{a}_{t-1} + u_t^a \quad (\text{A-12})$$



$$\hat{z}_t = \rho_z \hat{z}_{t-1} + u_t^z \quad (\text{A-13})$$

The pricing equation implies that

$$E_{t-1} \sum_{s=t}^{\infty} (\beta\alpha)^{s-t} \left[(\hat{p}_t^* - \hat{\Pi}_{t,s}) - (\hat{w}_s - \hat{a}_s) \right] = 0$$

or

$$\begin{aligned} \hat{p}_t^* &= (1 - \beta\alpha) E_{t-1} \sum_{s=t}^{\infty} (\beta\alpha)^{s-t} (\hat{w}_s - \hat{a}_s + \hat{\Pi}_{t,s}) \\ &= (1 - \beta\alpha) E_{t-1} [\hat{w}_t - \hat{a}_t + \hat{\Pi}_t] + (1 - \beta\alpha) E_{t-1} \sum_{s=t+1}^{\infty} (\beta\alpha)^{s-t} (\hat{w}_s - \hat{a}_s + \hat{\Pi}_{t,s}) \\ &= (1 - \beta\alpha) E_{t-1} [\hat{w}_t - \hat{a}_t + \hat{\Pi}_t] + \beta\alpha (1 - \beta\alpha) E_{t-1} \sum_{s=t+1}^{\infty} (\beta\alpha)^{s-(t+1)} (\hat{w}_s - \hat{a}_s + \hat{\Pi}_{t+1,s} + \hat{\Pi}_t) \\ &= (1 - \beta\alpha) E_{t-1} [\hat{w}_t - \hat{a}_t] + E_{t-1} \hat{\Pi}_t + \beta\alpha E_{t-1} \hat{p}_{t+1}^* \end{aligned}$$

where the final equality makes use of the law of iterated conditional expectations. Linearising the expression for the aggregate price level gives:

$$\begin{aligned} 1 &= \alpha \left(\frac{1}{z_t \Pi_t} \right)^{\frac{\mu-1}{\mu}} + (1 - \alpha) \left(\frac{p_t^*}{z_t \Pi_t} \right)^{\frac{\mu-1}{\mu}} \\ 0 &= -\alpha \left(\hat{\Pi}_t + \hat{z}_t \right) + (1 - \alpha) \left[\hat{p}_t^* - \hat{\Pi}_t - \hat{z}_t \right] \end{aligned}$$

so that

$$\hat{\Pi}_t = (1 - \alpha) \hat{p}_t^* - \hat{z}_t$$

Using this information in the log-linearised pricing equation gives:

$$\hat{\Pi}_t = (1 - \alpha) (1 - \beta\alpha) E_{t-1} [\hat{w}_t - \hat{a}_t] + (1 - \alpha) E_{t-1} \tilde{\Pi}_t + \alpha\beta E_{t-1} \hat{\Pi}_{t+1} - \rho_z (1 - \alpha\beta\rho_z) z_{t-1} - u_t^z \quad (\text{A-14})$$

which is a version of the New Keynesian Phillips curve. Note that here we have substituted out for the expectations (based on date $t - 1$ information) of the z shock. Equations (A-6)–(A-14) can be solved for the nine variables in the model: $\{\hat{c}, \hat{h}, \hat{y}, \hat{R}, \hat{a}, \hat{g}, \hat{\Pi}, \hat{w}, \hat{z}\}$.²⁹

²⁹Solutions for other variables, such as net dividend payments can be derived recursively.

A.2 The model under non-rational expectations

Here we focus on how the decision rules of households and firms are affected by the use of non-rational expectations. We will linearise the model equations around the same steady state analysed above and many of the linearised equations will carry across from the rational expectations case.

A.2.1 Households

As noted in the main text, household $i \in (0, 1)$ solves:

$$\max \tilde{E}_{i,t-1} \sum_{s=t}^{\infty} \beta^{s-t} \left[\frac{c_{i,s}^{1-\sigma}}{1-\sigma} - \chi \frac{h_{i,s}^{1+\gamma}}{1+\gamma} \right]$$

subject to

$$b_{i,s} - \frac{R_{s-1}}{\Pi_s} b_{i,s-1} - w_s h_{i,s} - d_s + c_{i,s} = 0$$

The Lagrangean for the problem is formed in the same way as in the rational expectations version and gives rise to the following first-order conditions for c_i , h_i and b_i :

$$\begin{aligned} \tilde{E}_{i,t-1} [c_{i,s}^{-\sigma} - \lambda_{i,s}] &= 0 \\ \tilde{E}_{i,t-1} [-\chi h_{i,s}^{\gamma} + \lambda_{i,s} w_s] &= 0 \\ \tilde{E}_{i,t-1} \left[-\lambda_{i,s} + \frac{\beta R_s}{\Pi_{s+1}} \lambda_{i,s+1} \right] &= 0 \end{aligned}$$

for $s = t, t + 1, \dots$. The only difference from the rational expectations case at this stage is that the expectations operator $\tilde{E}_{i,t-1}$ replaces the rational expectations operator E_{t-1} .

To proceed, we first log-linearise the equations around the steady state. As before $\hat{x}_t \equiv \ln(x_t/x)$ denotes the log deviation of x_t from its steady-state level x . We also define $\tilde{x}_t \equiv (x_t - x)$ as the absolute difference of x_t from its steady state. The latter will be used for bond holdings, since we linearise around a steady state in which bond market clearing implies $b = 0$. Linearising gives:

$$\begin{pmatrix} \tilde{b}_{i,s} - \beta^{-1} \tilde{b}_{i,s-1} - (1 - \mu) (\hat{w}_s + \hat{h}_{i,s}) \\ -(\mu - \psi_g) \hat{d}_s + (1 - \psi_g) \hat{c}_{i,s} \end{pmatrix} = 0 \quad (\text{A-15})$$

$$\tilde{E}_{i,t-1} [-\sigma \hat{c}_{i,s} - \hat{\lambda}_{i,s}] = 0 \quad (\text{A-16})$$

$$\tilde{E}_{i,t-1} [\gamma \hat{h}_{i,s} - \hat{w}_s - \hat{\lambda}_{i,s}] = 0 \quad (\text{A-17})$$

$$\tilde{E}_{i,t-1} \left[-\hat{\lambda}_{i,s} + \hat{\lambda}_{i,s+1} + \left(\hat{R}_s - \hat{\Pi}_{s+1} \right) \right] = 0 \quad (\text{A-18})$$

where we make use of the fact that in the steady state: $\beta R / \Pi = 1$; $w = 1 - \mu$; $h = 1$; $d = \mu - \psi_g$ and $c = 1 - \psi_g$.³⁰

To derive the decision rules, we begin with the budget constraint (A-15). We know that:

$$\begin{aligned} \tilde{E}_{i,t-1} \begin{bmatrix} \tilde{b}_{i,t} - \beta^{-1} \tilde{b}_{i,t-1} - (1 - \mu) (\hat{w}_t + \hat{h}_{i,t}) \\ - (\mu - \psi_g) \hat{d}_t + (1 - \psi_g) \hat{c}_{i,t} \end{bmatrix} &= 0 \\ \tilde{E}_{i,t-1} \begin{bmatrix} \tilde{b}_{i,t+1} - \beta^{-1} \tilde{b}_{i,t} - (1 - \mu) (\hat{w}_{t+1} + \hat{h}_{i,t+1}) \\ - (\mu - \psi_g) \hat{d}_{t+1} + (1 - \psi_g) \hat{c}_{i,t+1} \end{bmatrix} &= 0 \\ &\dots \end{aligned}$$

which means that we can combine the equations to give:

$$\tilde{E}_{i,t-1} \begin{bmatrix} \beta \begin{bmatrix} \tilde{b}_{i,t+1} - (1 - \mu) (\hat{w}_{t+1} + \hat{h}_{i,t+1}) \\ - (\mu - \psi_g) \hat{d}_{t+1} + (1 - \psi_g) \hat{c}_{i,t+1} \\ - \beta^{-1} \tilde{b}_{i,t-1} - (1 - \mu) (\hat{w}_t + \hat{h}_{i,t}) \\ - (\mu - \psi_g) \hat{d}_t + (1 - \psi_g) \hat{c}_{i,t} \end{bmatrix} \end{bmatrix} = 0$$

Repeated substitution in this fashion yields:

$$\tilde{E}_{i,t-1} \left[\lim_{s \rightarrow \infty} \beta^{s-t} \tilde{b}_{i,s+1} + \sum_{s=t}^{\infty} \beta^{s-t} \begin{pmatrix} (1 - \psi_g) \hat{c}_{i,s} - \beta^{-1} \tilde{b}_{i,t-1} - (\mu - \psi_g) \hat{d}_s \\ - (1 - \mu) (\hat{w}_s + \hat{h}_{i,s}) \end{pmatrix} \right] = 0$$

or

$$\begin{aligned} &\tilde{E}_{i,t-1} \sum_{s=t}^{\infty} \beta^{s-t} (1 - \psi_g) \hat{c}_{i,s} \quad (\text{A-19}) \\ &= \tilde{E}_{i,t-1} \sum_{s=t}^{\infty} \beta^{s-t} \begin{pmatrix} \beta^{-1} \tilde{b}_{i,t-1} + (\mu - \psi_g) \hat{d}_s \\ + (1 - \mu) (\hat{w}_s + \hat{h}_{i,s}) \end{pmatrix} \end{aligned}$$

where we have imposed the constraint:

$$\lim_{s \rightarrow \infty} \tilde{E}_{i,t-1} \beta^{s-t} \tilde{b}_{i,s+1} = 0$$

which is standard in the RE literature. In this context the assumption represents the assumption that the household's perceptions about wealth are in some sense bounded. Households are not

³⁰Note also that equation (A-15) contains a term representing the interest component of income from bond holdings (given by $-b\beta^{-1} (\hat{R}_{s-1} - \hat{\Pi}_s)$) which disappears when we linearise around the steady state in which $b = 0$.

allowed to believe that they can increase their borrowing faster than the financing requirements in the long run. Imposing this assumption means that we will be able to derive a consumption function from the conventional condition stating that the household's expected consumption stream exhausts the present value of their expected net income.

How strong is this assumption? Under rational expectations, conditions of this type will generally hold (at least along the equilibrium path if not under more general conditions). But there is no reason to assume that the behaviour of a household with arbitrary expectations over future events will satisfy this constraint. This means that our model effectively focuses attention on expectations schemes that have 'sensible' long-run properties. Moreover, in principle we should check on a case-by-case basis whether the expectations implied by the forecasting rules that we consider satisfy this constraint. We expect that general 'VAR' expectations of the form considered in Section 2.4 will satisfy this constraint as long as the eigenvalues in the projection matrices are not too large. It is not clear, however, that ruling out 'bubble' solutions in consumption decisions offers the best chance of generating strong expectational dynamics: laboratory experiments with subjects based on asset pricing models suggest that trend following behaviour can co-ordinate expectations on paths that look (locally) like explosive bubble solutions as shown by Anufriev and Hommes (2006).

To proceed, we combine **(A-16)** and **(A-17)** to give:

$$\tilde{E}_{i,t-1} h_{i,s} = \tilde{E}_{i,t-1} \left[\gamma^{-1} \hat{w}_s - \frac{\sigma}{\gamma} \hat{c}_{i,s} \right]$$

Using this information allows us to write **(A-19)** as:

$$\begin{aligned} & \tilde{E}_{i,t-1} \sum_{s=t}^{\infty} \beta^{s-t} \left[1 - \psi_g + \frac{\sigma(1-\mu)}{\gamma} \right] \hat{c}_{i,s} & \text{(A-20)} \\ = & \tilde{E}_{i,t-1} \sum_{s=t}^{\infty} \beta^{s-t} \begin{pmatrix} \beta^{-1} \tilde{b}_{i,t-1} + (\mu - \psi_g) \hat{d}_s \\ + (1-\mu)(1+\gamma^{-1}) \hat{w}_s \end{pmatrix} \end{aligned}$$

Combining **(A-16)** and **(A-18)** gives:

$$\tilde{E}_{i,t-1} \hat{c}_{i,t+1} = \tilde{E}_{i,t-1} \left[\hat{c}_{i,t} + \frac{1}{\sigma} \left(\hat{R}_t - \hat{\Pi}_{t+1} \right) \right]$$

so that in general

$$\begin{aligned}\tilde{E}_{i,t-1}\hat{c}_{i,s} &= \tilde{E}_{i,t-1}\left[\hat{c}_{i,t} + \frac{1}{\sigma}\sum_{k=t+1}^s\left(\hat{R}_{k-1} - \hat{\Pi}_k\right)\right] \\ &= \tilde{E}_{i,t-1}\left[\hat{c}_{i,t} + \frac{1}{\sigma}S_{t,s}^R\right]\end{aligned}$$

for $s = t + 1, t + 2, \dots$ where

$$S_{t,s}^R \equiv \begin{cases} 0 & s = t \\ \sum_{i=t+1}^s\left(\hat{R}_{s-1} - \hat{\Pi}_s\right) & s = t + 1, t + 2, \dots \end{cases}$$

Putting this into (A-20) gives:

$$\begin{aligned}&\left[1 - \psi_g + \frac{\sigma(1-\mu)}{\gamma}\right]\tilde{E}_{i,t-1}\sum_{s=t}^{\infty}\beta^{s-t}\left[\hat{c}_{i,t} + \frac{1}{\sigma}S_{t,s}^R\right] \\ &= \tilde{E}_{i,t-1}\sum_{s=t}^{\infty}\beta^{s-t}\begin{pmatrix} (1-\mu)(1+\gamma^{-1})\hat{w}_s \\ +\beta^{-1}\tilde{b}_{i,t-1} + (\mu - \psi_g)\hat{d}_s \end{pmatrix}\end{aligned}$$

which can be written as:

$$\begin{aligned}\hat{c}_{i,t} &= \frac{1-\beta}{1-\psi_g + \frac{\sigma(1-\mu)}{\gamma}}\tilde{E}_{i,t-1}\sum_{s=t}^{\infty}\beta^{s-t}\begin{pmatrix} (1-\mu)(1+\gamma^{-1})\hat{w}_s \\ +\beta^{-1}\tilde{b}_{i,t-1} + (\mu - \psi_g)\hat{d}_s \end{pmatrix} \\ &\quad - \frac{1-\beta}{\sigma}\tilde{E}_{i,t-1}\sum_{s=t+1}^{\infty}\beta^{s-t}S_{t,s}^R\end{aligned}$$

To complete the description of the consumption equation, we need to unpack the final term:

$$\begin{aligned}\tilde{E}_{i,t-1}\sum_{s=t+1}^{\infty}\beta^{s-t}S_{t,s}^R &= \tilde{E}_{i,t-1}\sum_{s=t+1}^{\infty}\beta^{s-t}\sum_{k=t+1}^s\left(\hat{R}_{k-1} - \hat{\Pi}_k\right) \\ &= \tilde{E}_{i,t-1}\left[\begin{aligned} &\beta\left(\hat{R}_t - \hat{\Pi}_{t+1}\right) + \\ &\beta^2\left(\hat{R}_t - \hat{\Pi}_{t+1}\right) + \beta^2\left(\hat{R}_{t+1} - \hat{\Pi}_{t+2}\right) + \\ &\beta^3\left(\hat{R}_t - \hat{\Pi}_{t+1}\right) + \beta^3\left(\hat{R}_{t+1} - \hat{\Pi}_{t+2}\right) + \beta^3\left(\hat{R}_{t+2} - \hat{\Pi}_{t+3}\right) + \\ &\dots \end{aligned}\right] \\ &= \tilde{E}_{i,t-1}\sum_{s=t+1}^{\infty}\frac{\beta^{s-t}}{1-\beta}\left(\hat{R}_{s-1} - \hat{\Pi}_s\right)\end{aligned}$$

so that the consumption equation can be written:

$$\hat{c}_{i,t} = \frac{1 - \beta}{1 - \psi_g + \frac{\sigma(1-\mu)}{\gamma}} \tilde{E}_{i,t-1} \sum_{s=t}^{\infty} \beta^{s-t} \left(\begin{array}{l} (1 - \mu) (1 + \gamma^{-1}) \hat{w}_s \\ + \beta^{-1} \tilde{b}_{i,t-1} + (\mu - \psi_g) \hat{a}_s \end{array} \right) - \frac{\beta}{\sigma} \tilde{E}_{i,t-1} \sum_{s=t}^{\infty} \beta^{s-t} (\hat{R}_s - \hat{\Pi}_{s+1})$$

A.2.2 Firms

The environment faced by firms is essentially the same as that described in Section 2.1.2. To analyse their optimal pricing decision, we begin with a version of the pricing equation (A-4) derived in Section 2.1.2:

$$\bar{E}_{j,t-1} \sum_{s=t}^{\infty} \Lambda_s (\beta\alpha)^{s-t} \left(\frac{\mu - 1}{\mu} \frac{p_{j,t}}{\Pi_{t,s}} + \frac{1}{\mu} \frac{w_s}{a_s} \right) \left(\frac{p_{j,t}}{\Pi_{t,s}} \right)^{-\frac{1}{\mu}} z_s^{\frac{1-\mu}{\mu}} y_s = 0$$

where in this case the only difference from equation (A-4) is that the expectation of firm j is denoted as the (non-rational) expectation $\bar{E}_{j,t-1}$.

Log-linearising around the steady state gives:

$$\bar{E}_{j,t-1} \sum_{s=t}^{\infty} (\beta\alpha)^{s-t} \left[(\hat{p}_{j,t} - \hat{\Pi}_{t,s}) - (\hat{w}_s - \hat{a}_s) \right] = 0$$

or

$$\hat{p}_{j,t} = (1 - \beta\alpha) \bar{E}_{j,t-1} \sum_{s=t}^{\infty} (\beta\alpha)^{s-t} (\hat{w}_s - \hat{a}_s + \hat{\Pi}_{t,s})$$

To complete the derivation of the optimal relative price, consider the final term:

$$\begin{aligned} \bar{E}_{j,t-1} \sum_{s=t}^{\infty} (\beta\alpha)^{s-t} \hat{\Pi}_{t,s} &= \bar{E}_{j,t-1} \left[\begin{array}{l} \hat{\Pi}_t + \\ \beta\alpha \hat{\Pi}_t + \beta\alpha \hat{\Pi}_{t+1} + \\ (\beta\alpha)^2 \hat{\Pi}_t + (\beta\alpha)^2 \hat{\Pi}_{t+1} + (\beta\alpha)^2 \hat{\Pi}_{t+2} + \\ \dots \end{array} \right] \\ &= \bar{E}_{j,t-1} \left[\begin{array}{l} \hat{\Pi}_t (1 + \beta\alpha + (\beta\alpha)^2 + \dots) + \\ \hat{\Pi}_{t+1} (\beta\alpha + (\beta\alpha)^2 + (\beta\alpha)^3 + \dots) + \\ \hat{\Pi}_{t+2} ((\beta\alpha)^2 + (\beta\alpha)^3 + (\beta\alpha)^4 + \dots) + \dots \end{array} \right] \\ &= \bar{E}_{j,t-1} \sum_{s=t}^{\infty} \frac{(\beta\alpha)^{s-t}}{1 - \beta\alpha} \hat{\Pi}_s \end{aligned}$$

so that the optimal relative price can be written as:

$$\hat{p}_{j,t} = (1 - \beta\alpha) \bar{E}_{j,t-1} \sum_{s=t}^{\infty} (\beta\alpha)^{s-t} (\hat{w}_s - \hat{a}_s) + \bar{E}_{j,t-1} \sum_{s=t}^{\infty} (\beta\alpha)^{s-t} \hat{\Pi}_s$$

A.2.3 Aggregation

As explained in the text, we assume that there are a finite set of ‘predictors’ available to each group of agents. From this point on we will use the index i (j) to denote the decisions of a household (firm) using predictor $i \in \{1, \dots, I\}$ ($j \in \{1, \dots, J\}$). The mass of households (firms) using predictor i (j) is given by n_i (m_j) where

$$\sum_{i=1}^I n_i = \sum_{j=1}^J m_j = 1$$

This means that the model equations can be written as follows. The average reset price of firms is

$$\begin{aligned} \hat{p}_t^* &= \sum_{j=1}^J m_j \hat{p}_{j,t} \\ &= (1 - \beta\alpha) \sum_{j=1}^J m_j \bar{E}_{j,t-1} \sum_{s=t}^{\infty} (\beta\alpha)^{s-t} (\hat{w}_s - \hat{a}_s) + \sum_{j=1}^J m_j \bar{E}_{j,t-1} \sum_{s=t}^{\infty} (\beta\alpha)^{s-t} \hat{\Pi}_s \end{aligned}$$

where, for each $j \in \{1, \dots, J\}$, m_j is the fraction of firms setting price \hat{p}_j .

Inflation is given by:

$$\hat{\Pi}_t = (1 - \alpha) \hat{p}_t^* - \hat{z}_t$$

The linearised resource constraint and monetary policy rule are the same as in the rational expectations model:

$$\begin{aligned} \hat{y}_t &= (1 - \psi_g) \hat{c}_t + \psi_g \hat{g}_t \\ \hat{R}_t &= (1 - \phi_r) (\phi_\pi \hat{\Pi}_{t-1} + \phi_y \hat{y}_{t-1}) + \phi_r \hat{R}_{t-1} + u_t^R \end{aligned}$$

The average level of consumption is:

$$\begin{aligned} \hat{c}_t &= \sum_{i=1}^I n_i \hat{c}_{i,t} \\ &= \frac{1-\beta}{1-\psi_g + \frac{\sigma(1-\mu)}{\gamma}} \sum_{i=1}^I n_i \tilde{E}_{i,t-1} \sum_{s=t}^{\infty} \beta^{s-t} \left[(1 - \mu) (1 + \gamma^{-1}) \hat{w}_s + (\mu - \psi_g) \hat{a}_s \right] \\ &\quad - \frac{\beta}{\sigma} \sum_{i=1}^I n_i \tilde{E}_{i,t-1} \sum_{s=t+1}^{\infty} \beta^{s-t} (\hat{R}_s - \hat{\Pi}_{s+1}) \end{aligned}$$

where we use the fact that bond market clearing requires:

$$\sum_{i=1}^I n_i \tilde{b}_{i,s} = 0$$



for all s .

Turning to the labour market, we note that our assumptions about the worker/shopper behaviour of the household gives:

$$\hat{w}_t = \gamma \hat{h}_t + \sigma \hat{c}_t$$

where we exploit the log-linear form of the labour supply choice of the household's 'worker'.

Noting that $\hat{h}_t = \hat{y}_t - \hat{a}_t - \hat{z}_t = (1 - \psi_g) \hat{c}_t + \psi_g \hat{g}_t - \hat{a}_t - \hat{z}_t$ we have:

$$\hat{w}_t = [\gamma (1 - \psi_g) + \sigma] \hat{c}_t + \gamma \psi_g \hat{g}_t - \gamma \hat{a}_t - \gamma \hat{z}_t$$

In this version of the model we need to solve out for net dividends since households must forecast this variable when making their consumption plans. We can see that the definition of dividends and the market clearing conditions imply:

$$d_t = \Delta_t - \tau_t$$

in equilibrium this is given by:

$$d_t = y_t - w_t h_t - g_t$$

which means that

$$\hat{d}_t = \frac{(1 - \psi_g) \mu}{\mu - \psi_g} \hat{c}_t - \frac{1 - \mu}{\mu - \psi_g} \hat{w}_t - \frac{(1 - \mu) \psi_g}{\mu - \psi_g} \hat{g}_t + \frac{1 - \mu}{\mu - \psi_g} \hat{a}_t + \frac{1 - \mu}{\mu - \psi_g} \hat{z}_t$$

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